Stochastic rainfall seasonality: Estimation and applications

Trevor Carey-Smith[1], Peter Thomson[2] & John Sansom[1]

1. National Institute of Water and Atmosphere, NZ

2. Statistics Research Associates Ltd, NZ

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Christchurch set for early start to spring

MYLES HUME

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Your Weather

Spring to be ruffled by strong winds
Burst pipe delays Riccarton traffic
Spring set to be a wet one
Christchurch set for early start to spring
Enjoy the sun before weather packs in
Snow warnings for alpine passes
Showers to interrupt winter sunshine
Fine weekend ahead as chilly winds

Christchurch is set for a string of fine weather as the beginning of spring looms.

With just one week left of what has been a cool and dry winter, the city looks to be in store for high cloud and light winds.

Today, MetService says Christchurch is set for cloud with fine spells this afternoon, looking to reach a high of 11 degrees Celsius.Fine weather

With temperatures peaking on Wednesday at 14C, the

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Christchurch property
Capsized fisherman t
New sports centre at
Work and Income to
Driver chased thief, w

Spring gusts in a bit early this year

By Harrison Christian

2:00 PM Monday Aug 18, 2014

Snow on the hills at Te Pohue off State Highway 5, near Napier, on Friday. Photo / Paul Taylor

The howling winds which ruffled the region last week, gusting up to 130km/h, are a sign of early spring, a forecaster says - and there are more to come.

WeatherWatch.co.nz forecaster Philip Duncan said powerful winds on Thursday and Friday were due to a "big pressure gradient over New Zealand".

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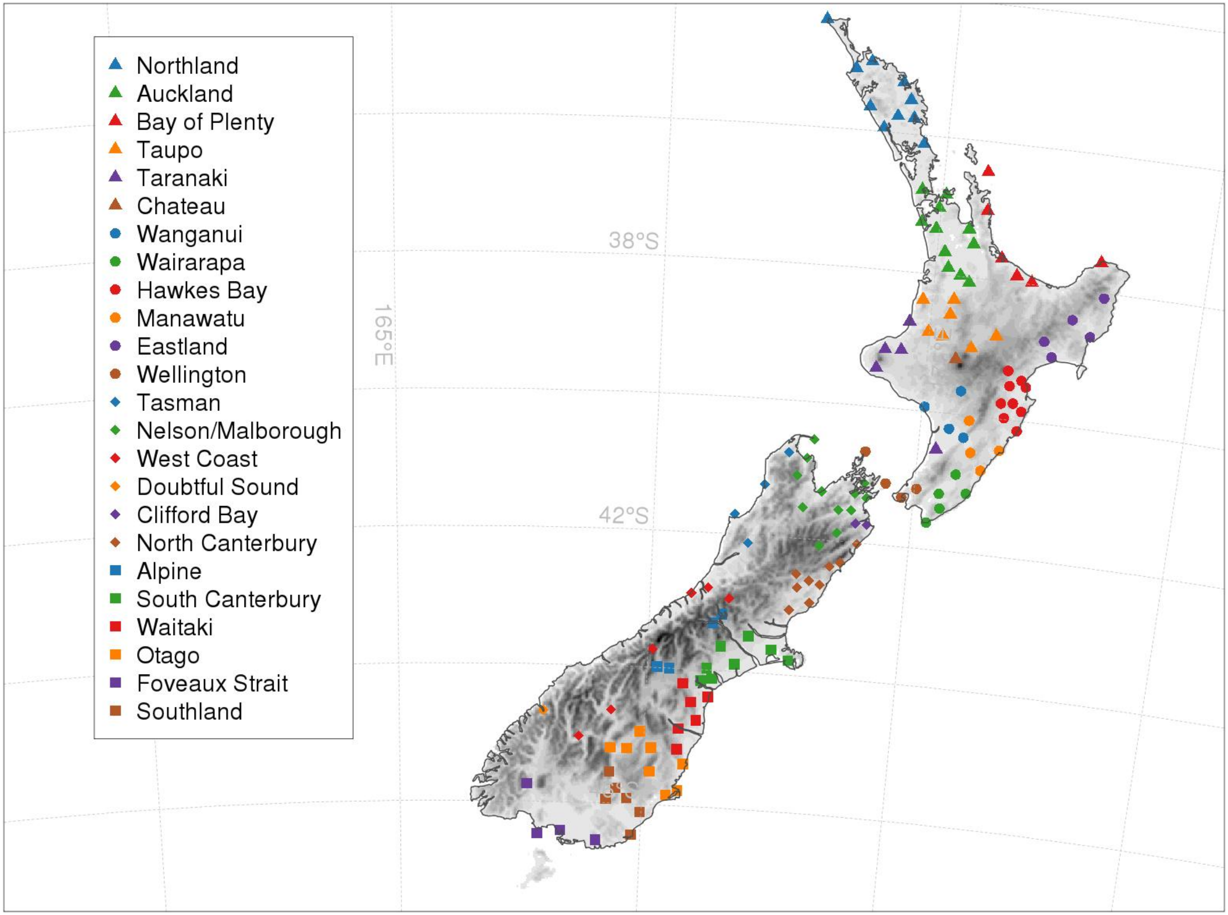
Stochastic seasonality

Seasons are not consistently deterministic.

- Stochastic seasonal variation accommodates more intra-annual variability
- A stochastic model for seasonality can be used to describe observed seasonal variability
- Conditional seasonality can be used to improve prediction


Northland
Auckland
Bay of Plenty
Taupo
Taranaki
Chateau
Wanganui
Wairarapa
Hawkes Bay
Manawatu
Eastland
Wellington
Tasman
Nelson/Malborough
West Coast
Doubtful Sound
Clifford Bay
North Canterbury
Alpine
South Canterbury
Waitaki
Otago
Foveaux Strait
Southland
38°S
42°S
165°E

Within season rainfall model

Based on Wilks (1998) and Thompson *et al.* (2007)

This is a (partially) hidden three state model:

- $S_t(k) = 0$ dry
- $S_t(k) = 1$ light rain
- $S_t(k) = 2$ heavy rain

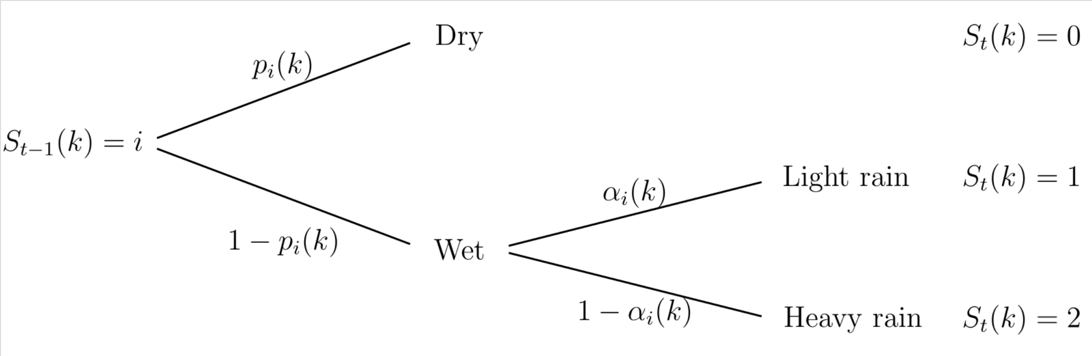
The rainfall for each site k and time t is

$$R_t(k) = \beta_{S_t(k)}(k) X_t(k)$$

where the $X_t(k)$ are temporally independent unit exponentials.

Six (dynamic) parameters per site:

$$p_i(k) = P(S_t(k) = 0 | S_{t-1}(k) = i)$$
$$\alpha_i(k) = P(S_t(k) = 1 | S_t(k) \neq 0, S_{t-1}(k) = i)$$
$$(i = 0, 1, 2)$$



Spatial dependence is built into $R_t(k)$ through:

$$X_t(k) = -\log(\mathbf{\Phi}(V_t(k)))$$

and $S_t(k)$ using:

$$S_t(k) = \begin{cases} 0 & U_t(k) \leq a_i(k) \\ 1 & a_i(k) < U_t(k) \leq b_i(k) \\ 2 & U_t(k) > b_i(k) \end{cases}$$

where

$$a_i(k) = \mathbf{\Phi}^{-1}(p_i(k))$$
$$b_i(k) = \mathbf{\Phi}^{-1}(p_i(k) + \alpha_i(k)(1 - p_i(k)))$$

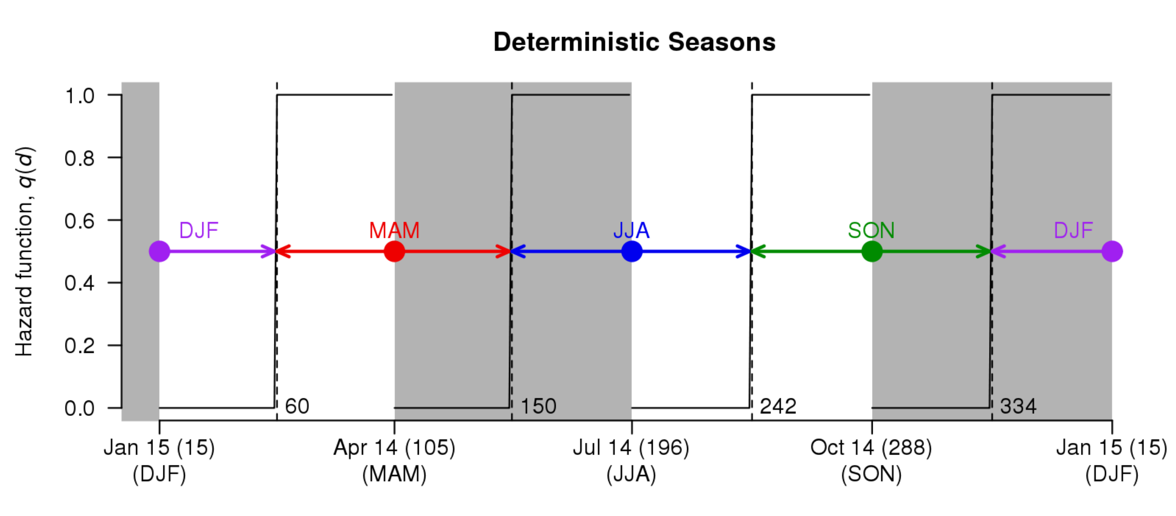
$\mathbf{U}_t$ and $\mathbf{V}_t$ are Gaussian processes based on separate correlation matrices, $\mathbf{\Omega}$ and $\mathbf{\Psi}$

Season switching model

- Variable season onset dates
- Variable season lengths
- Seasons strictly sequential
- Cyclical seasons
- Seasons common to all sites in region

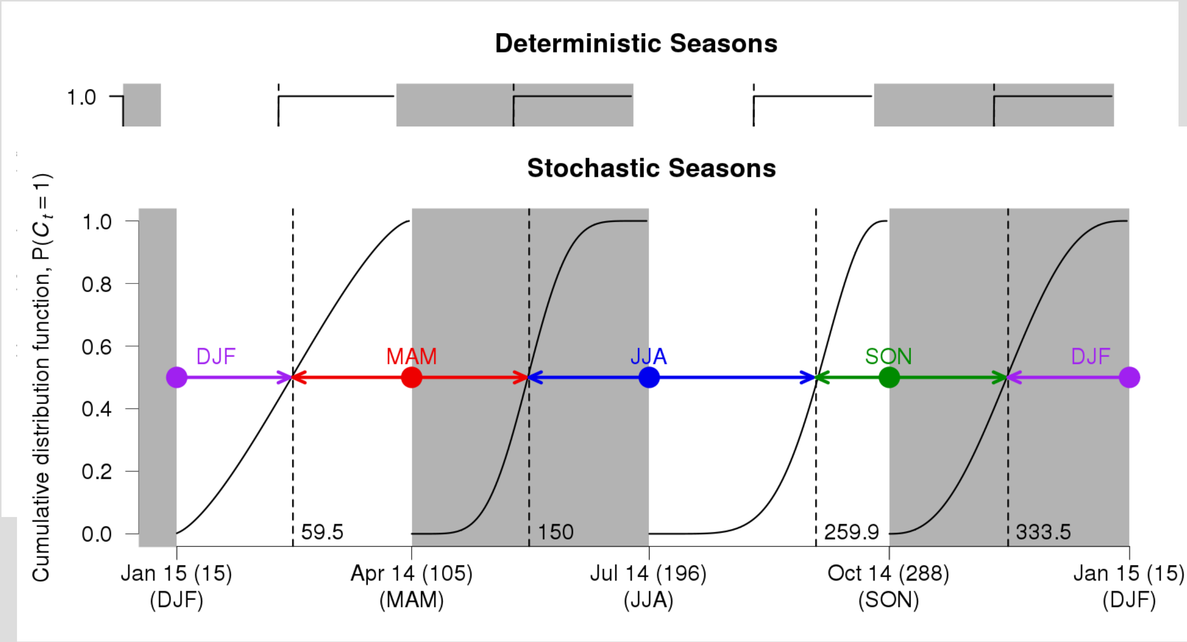
Daily rainfall is dynamically classified into homogeneous seasonal regimes with stochastic onsets and durations.

Traditionally defined seasons are replaced with "season change intervals" defined by "season anchor points".



Shading denotes season change intervals; colours denote seasons; filled circles denote season anchor points.

Traditionally defined seasons are replaced with “season change intervals” defined by “season anchor points”.



Shading denotes season change intervals; colours denote seasons; filled circles denote season anchor points.

Within any season change interval there are only **two possible seasons.**

The season change indictor

$$C_t = \begin{cases} 0 & \text{mover state} \\ 1 & \text{stayer} \end{cases}$$

is a Markov chain with transition probability matrix

$$\mathbf{Q}(d) = \begin{bmatrix} 1 - q(d) & q(d) \\ 0 & 1 \end{bmatrix}$$

Boundary conditions are also necessary to ensure $C_t = 0$ at the start and $C_t = 1$ at the end of the season change interval.

Examples of $q(d)$

This can be any discrete function. For example the hazard function, $q(d)$, can come from:

- a step function
- a Beta distribution
- a uniform onset distribution

The uniform onset function is useful for investigating seasonality within the data, but it is not useful for prediction.

Parameters of $q(d)$ can be estimated by maximising:

$$M = \sum_{t=2}^{T} \sum_{u=0}^{1} \sum_{v=0}^{1} \log \mathbf{Q}_{uv}(d_t) \gamma^{(uv)}(t)$$

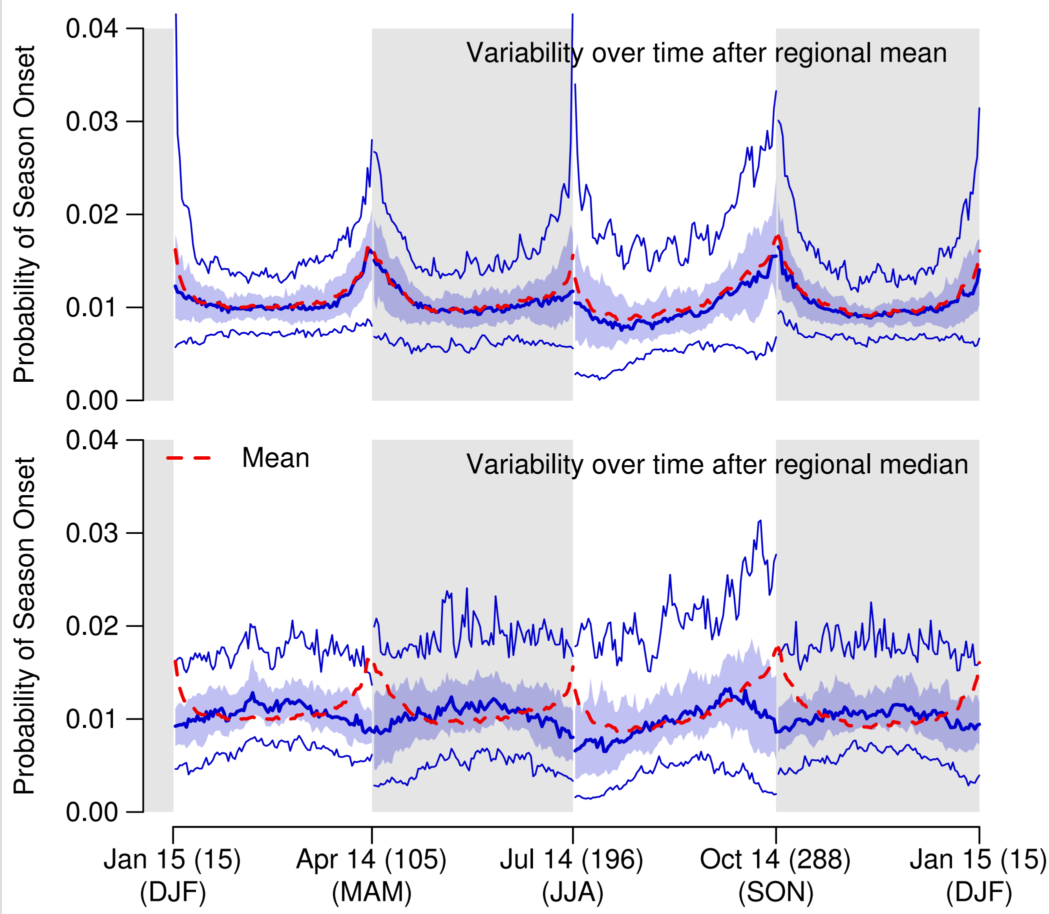
where

$$\gamma^{(uv)}(t) = P_0(C_{t-1} = u, C_t = v | \mathbf{R}).$$

This can be evaluated, but the cost is prohibitive if the number if sites is not small.

A simple predictor of $\gamma^{(uv)}(t)$ is

$$\tilde{\gamma}^{(uv)}(t) = \frac{1}{K} \sum_{k=1}^{K} P(C_{t-1} = u, C_t = v | \mathbf{R}(k)).$$



Variability over time after regional mean
Variability over time after regional median
Mean
Probability of Season Onset
0.00
0.01
0.02
0.03
0.04
Jan 15 (15)
(DJF)
Apr 14 (105)
(MAM)
Jul 14 (196)
(JJA)
Oct 14 (288)
(SON)
Jan 15 (15)
(DJF)

These methods of estimating $q(d)$, i.e.

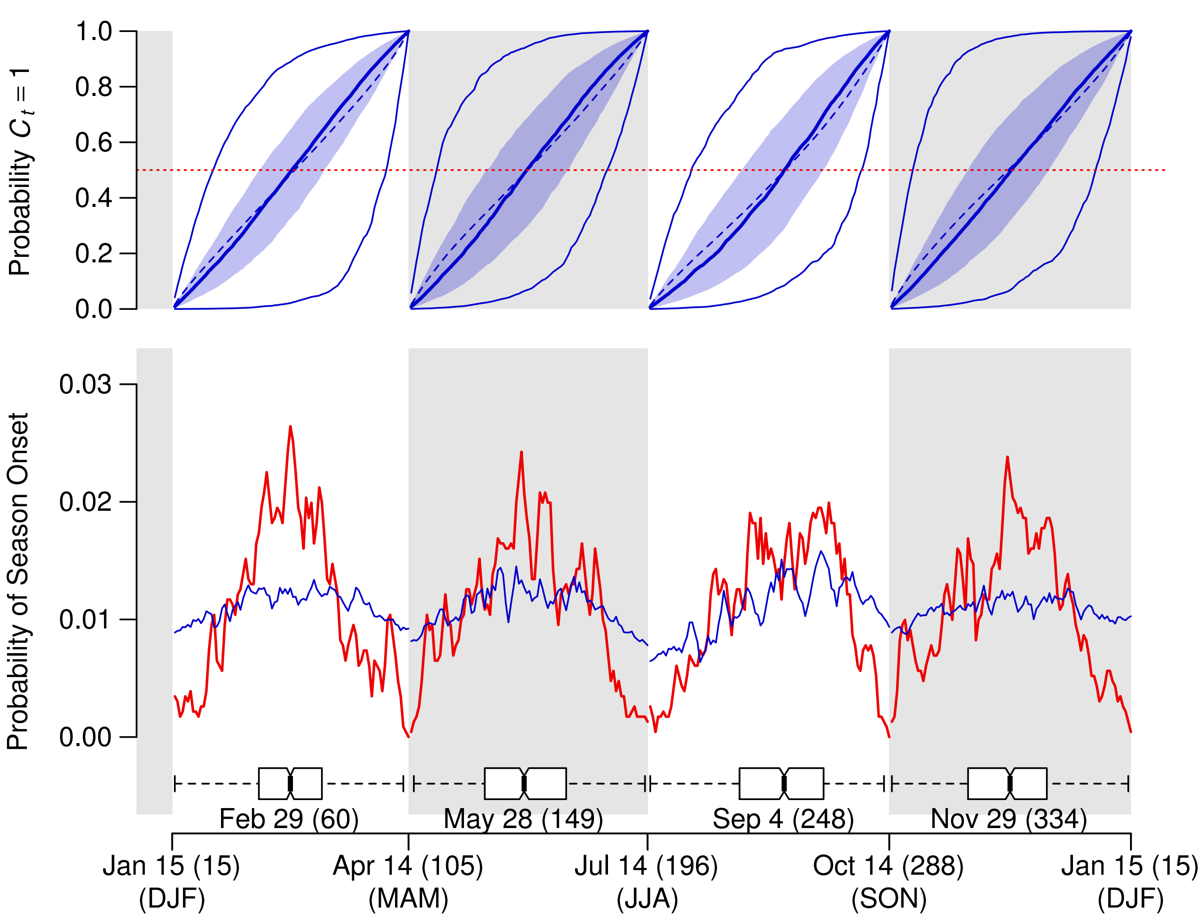
- **mean** over univariate estimates
- **median** over univariate estimates

both depend on combining the **daily probabilities** over space and time.

Another option is to use a

- **classification** rule

to choose the actual season change day for each site and season, then use the distribution of these days to characterise $q(d)$.



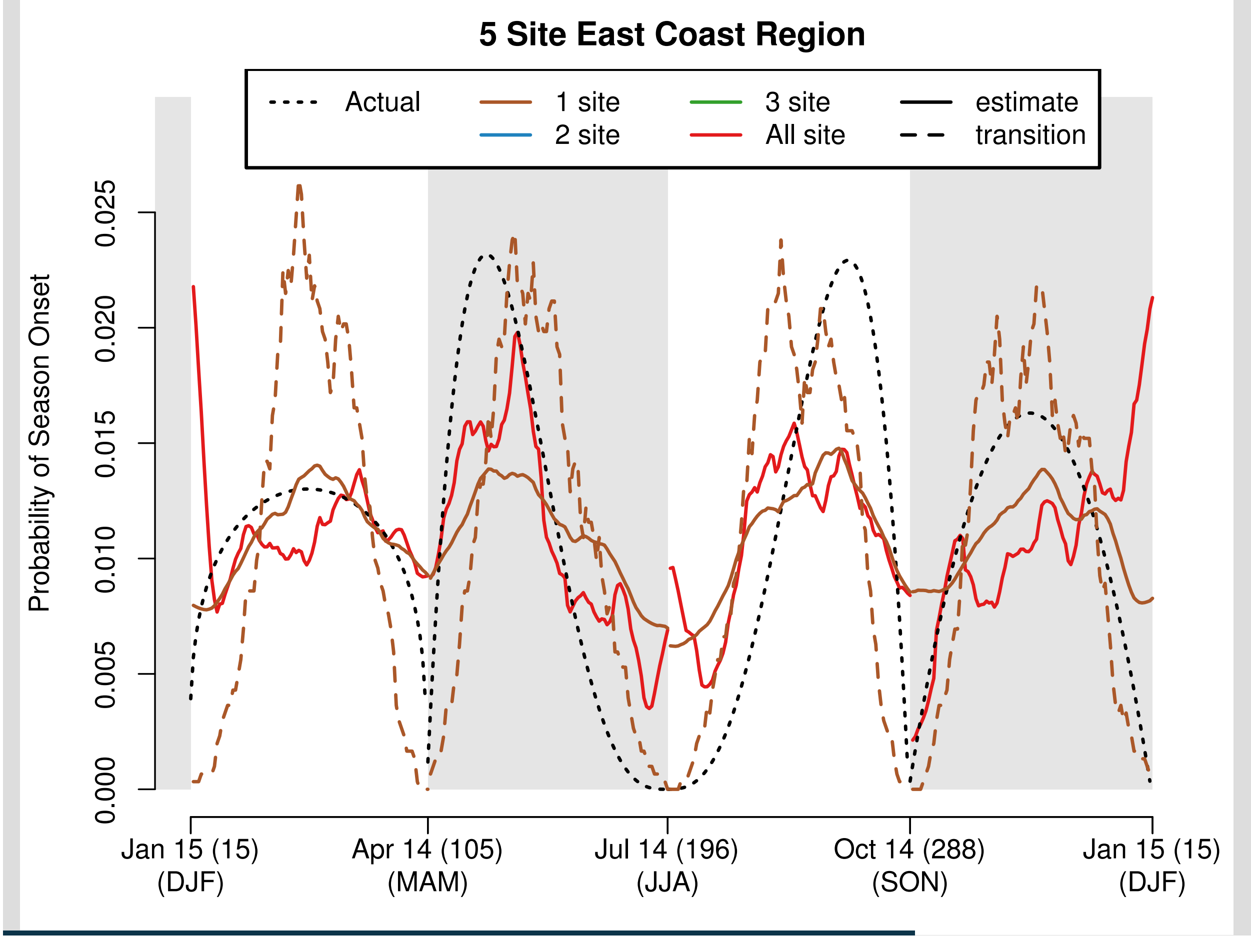
Probability $C_t = 1$
1.0
0.8
0.6
0.4
0.2
0.0
Probability of Season Onset
0.03
0.02
0.01
0.00
Feb 29 (60)
May 28 (149)
Sep 4 (248)
Nov 29 (334)
Jan 15 (15) (DJF)
Apr 14 (105) (MAM)
Jul 14 (196) (JJA)
Oct 14 (288) (SON)
Jan 15 (15) (DJF)

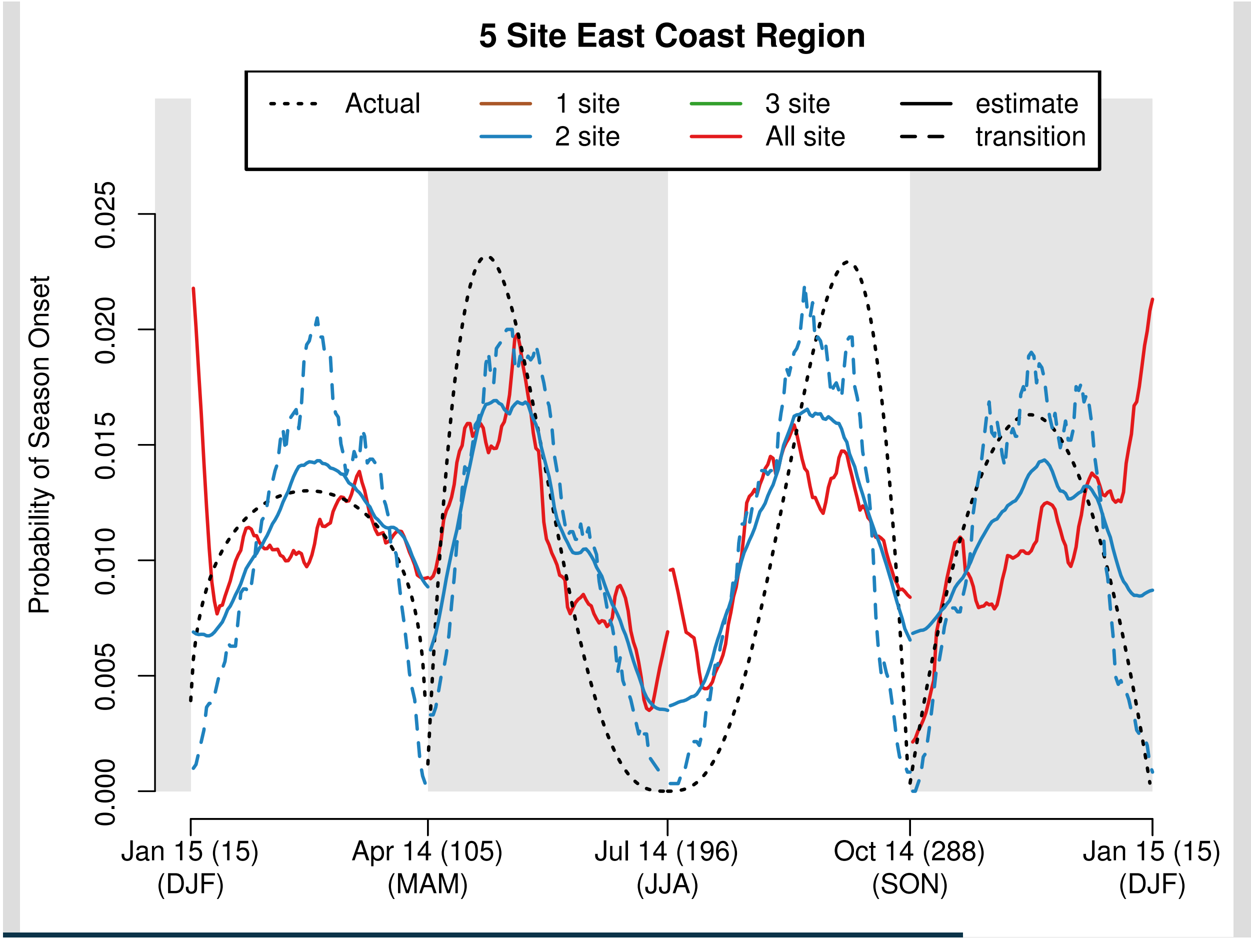
Another option, is to estimate the multi-site $\gamma^{(uv)}(t)$ by combining multiple **bivariate** or **trivariate** estimates.

- **mean** over univariate estimates
- **median** over univariate estimates
- **classification** rule
- combined **bivariate** or **trivariate** estimates

Different estimates can be compared to truth using simulated rainfall with known distribution of season onset dates.

By limiting the region to **5 sites**, $\gamma^{(uv)}(t)$ can be computed directly for comparison as well.


5 Site East Coast Region
Actual
1 site
2 site
3 site
All site
estimate
transition
Probability of Season Onset
0.000
0.005
0.010
0.015
0.020
0.025
Jan 15 (15)
(DJF)
Apr 14 (105)
(MAM)
Jul 14 (196)
(JJA)
Oct 14 (288)
(SON)
Jan 15 (15)
(DJF)



5 Site East Coast Region
Actual
1 site
2 site
3 site
All site
estimate
transition
Probability of Season Onset
0.000
0.005
0.010
0.015
0.020
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Jan 15 (15)
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Apr 14 (105)
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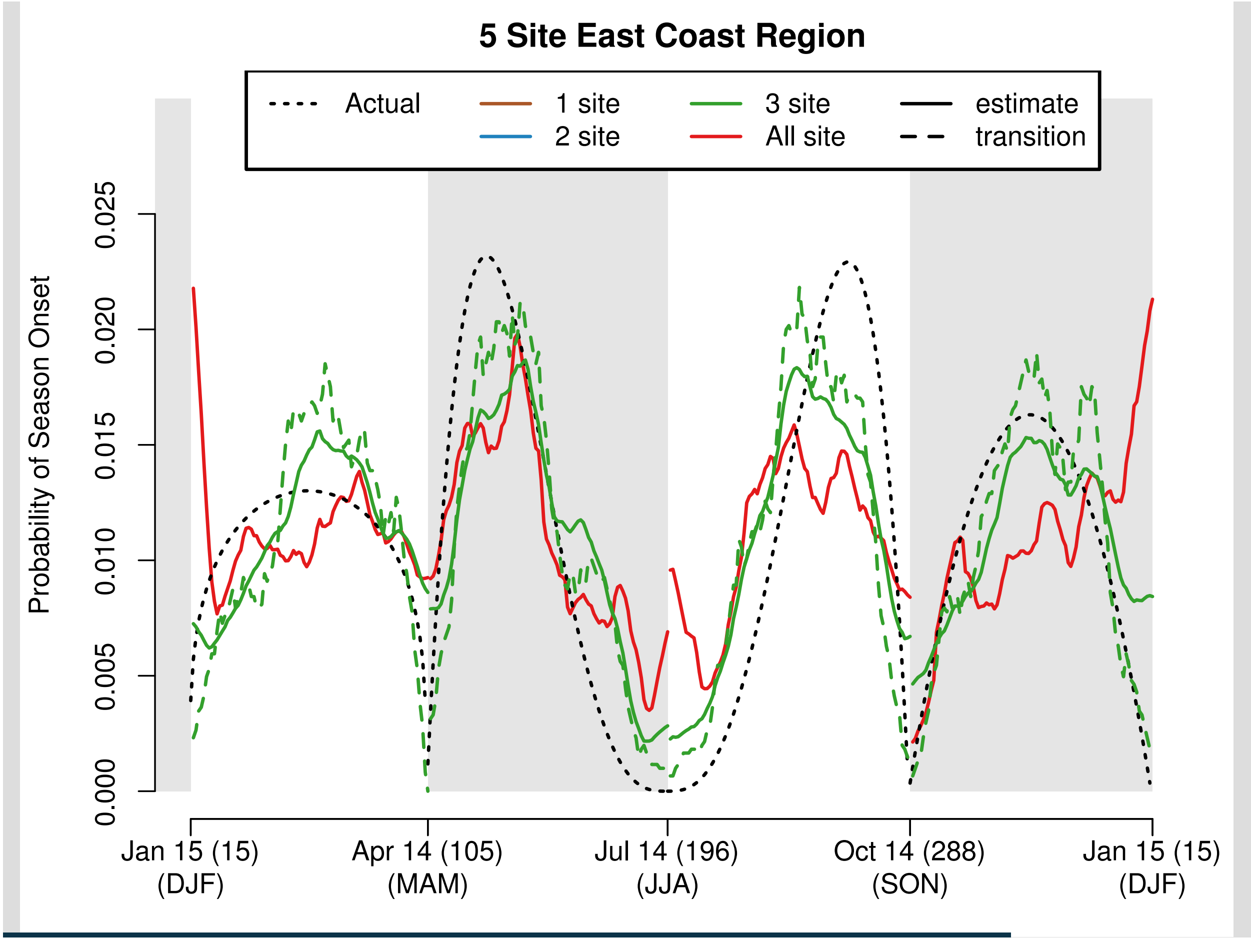
Jul 14 (196)
(JJA)
Oct 14 (288)
(SON)
Jan 15 (15)
(DJF)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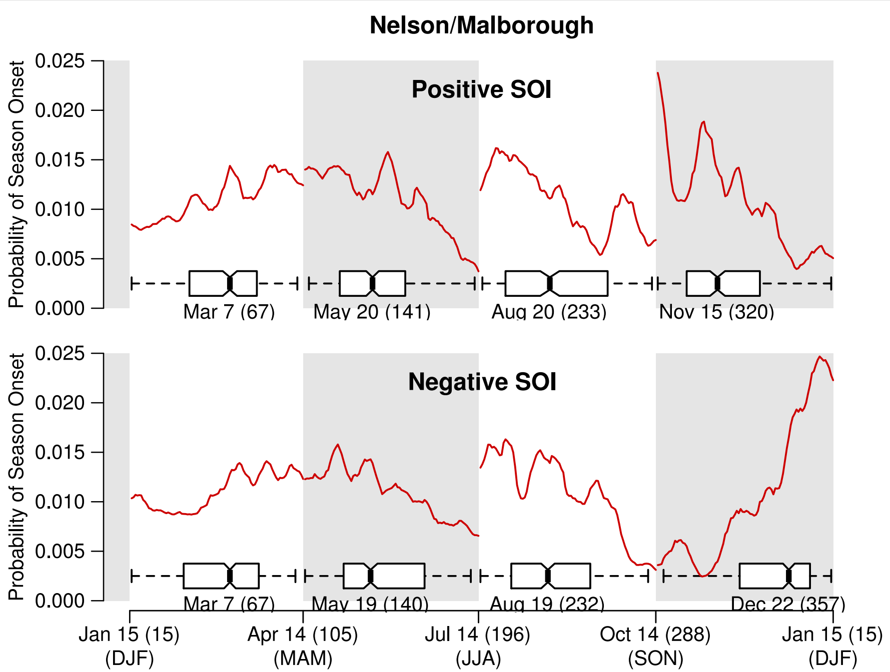
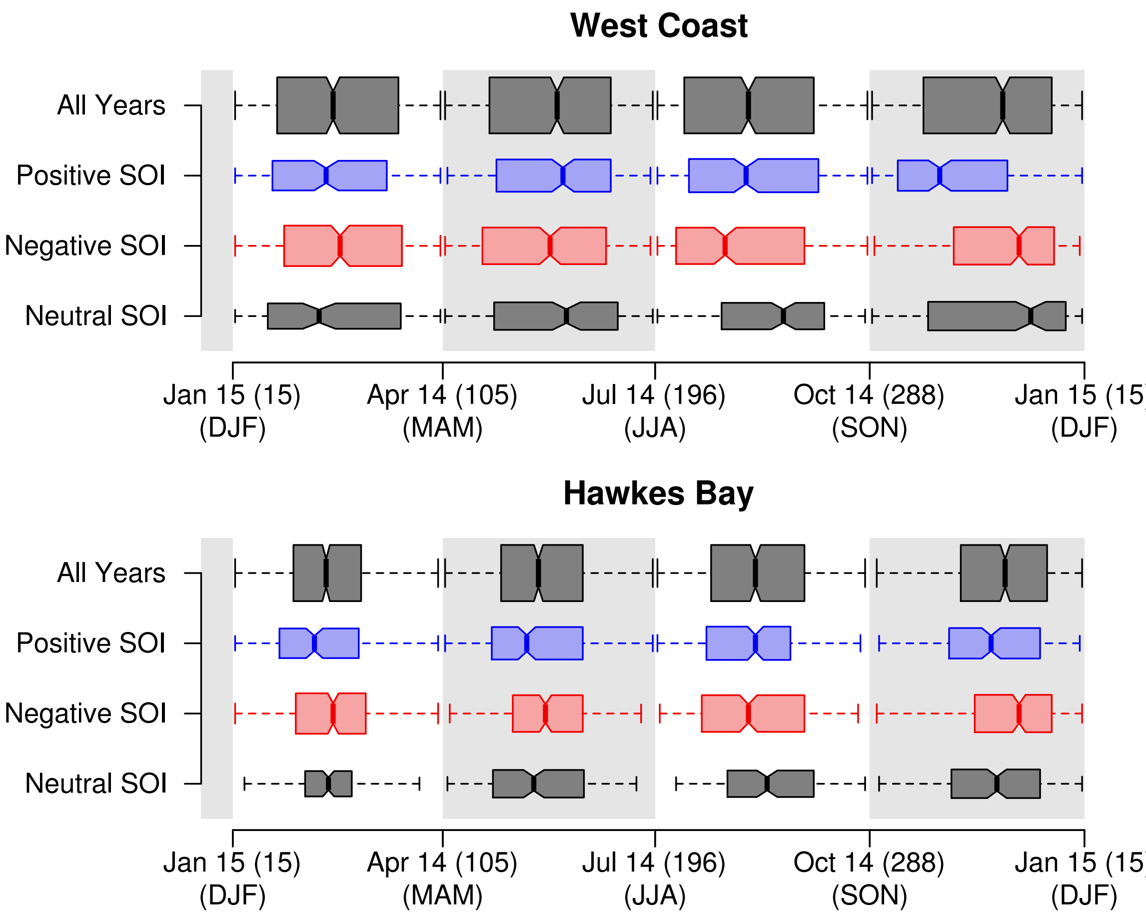
(MAM)
Jul 14 (196)
(JJA)
Oct 14 (288)
(SON)
Jan 15 (15)
(DJF)

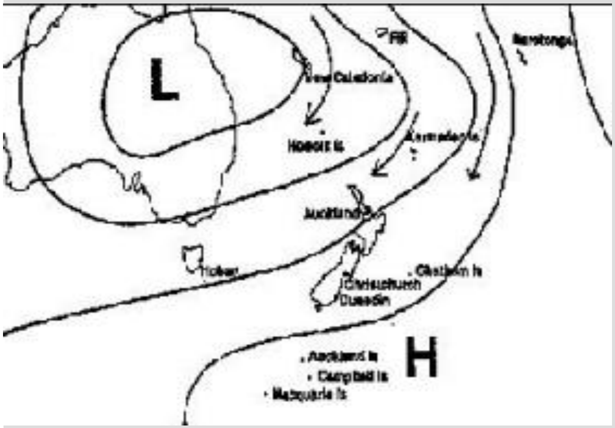
Climate drivers — El Niño

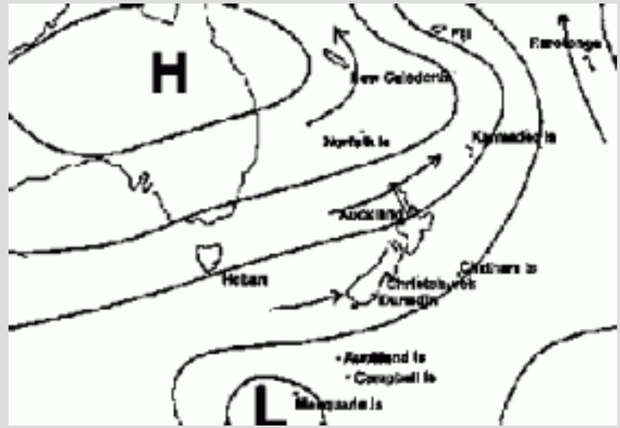
$q(d)$ can be categorised into regimes by conditioning on a climate index such as the SOI.

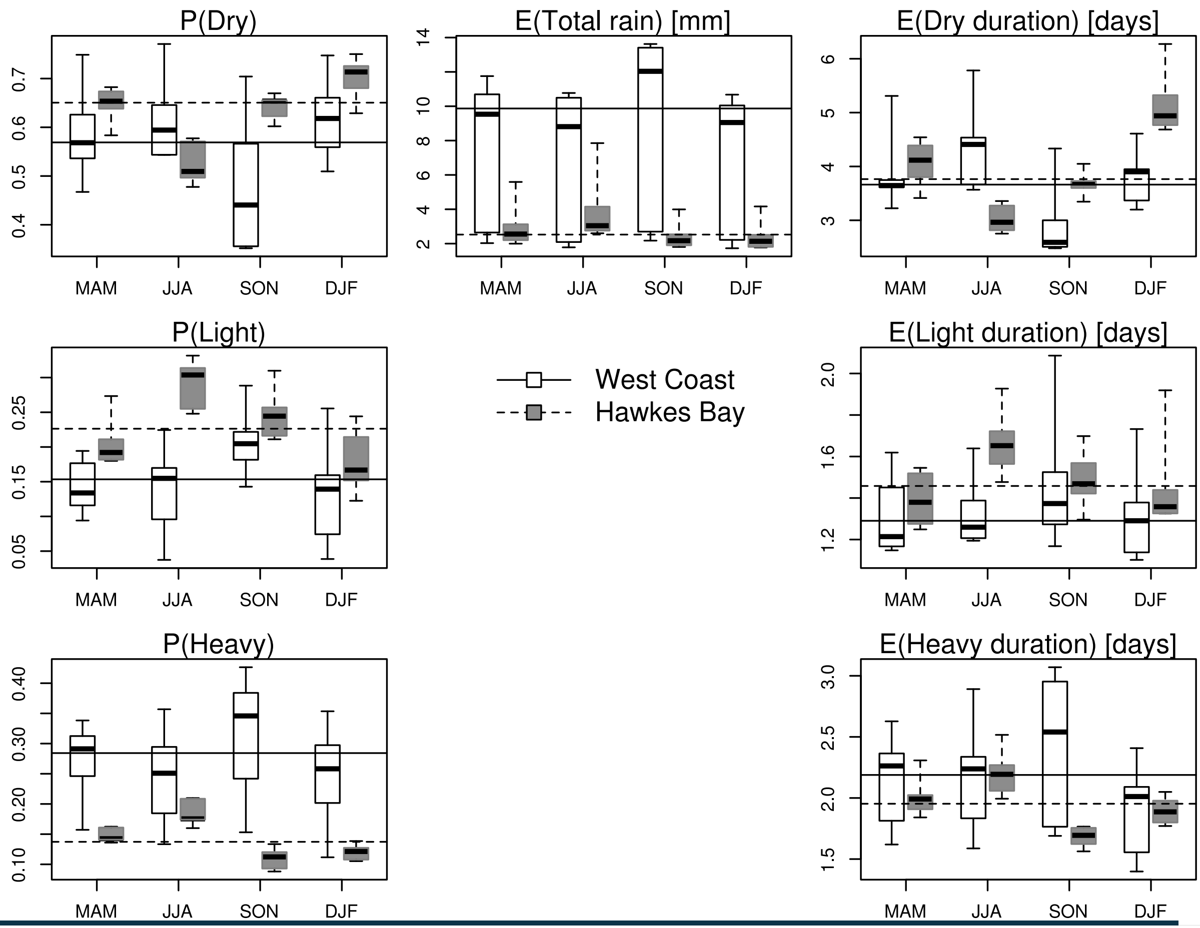



West Coast
All Years
Positive SOI
Negative SOI
Neutral SOI
Jan 15 (15) (DJF)
Apr 14 (105) (MAM)
Jul 14 (196) (JJA)
Oct 14 (288) (SON)
Jan 15 (15) (DJF)
Hawkes Bay
All Years
Positive SOI
Negative SOI
Neutral SOI
Jan 15 (15) (DJF)
Apr 14 (105) (MAM)
Jul 14 (196) (JJA)
Oct 14 (288) (SON)
Jan 15 (15) (DJF)

Typical La Niña and El Niño conditions







P(Dry)
E(Total rain) [mm]
E(Dry duration) [days]
P(Light)
E(Light duration) [days]
P(Heavy)
E(Heavy duration) [days]
West Coast
Hawkes Bay
MAM
JJA
SON
DJF

Summary

The stochastic seasonal model can be used as an investigative tool to:

- move beyond rigid or sinusoidal seasons
- tease out weak seasonality and easily characterise strong seasonality

Conditioning seasonality on climate indices could improve the predictibility of the rainfall model.

- Stochastic rainfall generation could be driven from short range seasonal outlooks
- An additional mechanism to improve climate model downscaling