Point processes- - abstraction and practical relevance

Janine Illian

University of St Andrews, Scotland

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some background – my interests

spatial statistics, in particular,
spatial and spatio-temporal point process modelling

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- *practically relevant*
- *realistically complex and*
- *accessible*

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perspective: applications in ecology and beyond

spatial point processes

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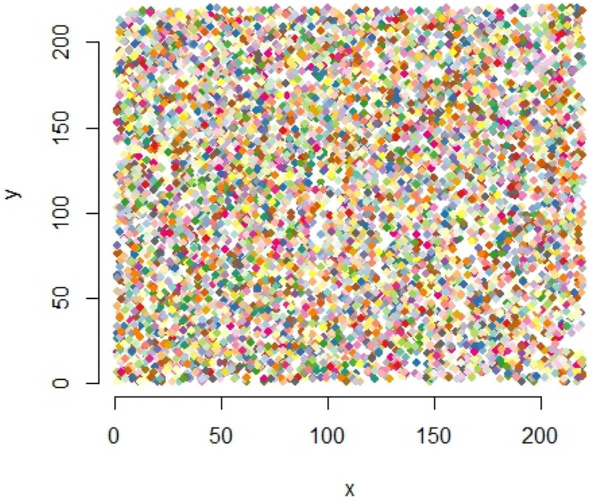

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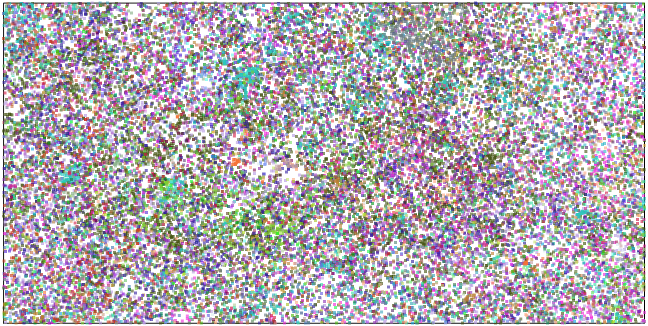

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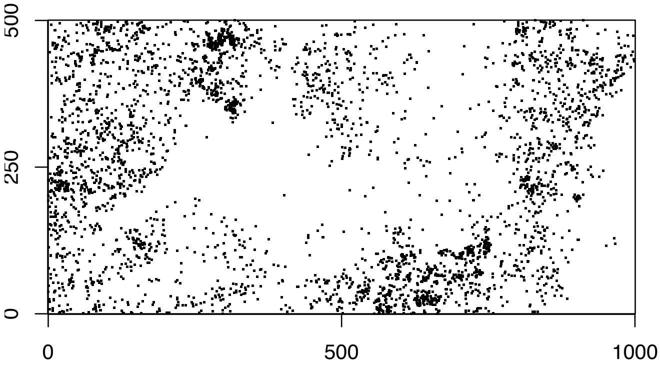

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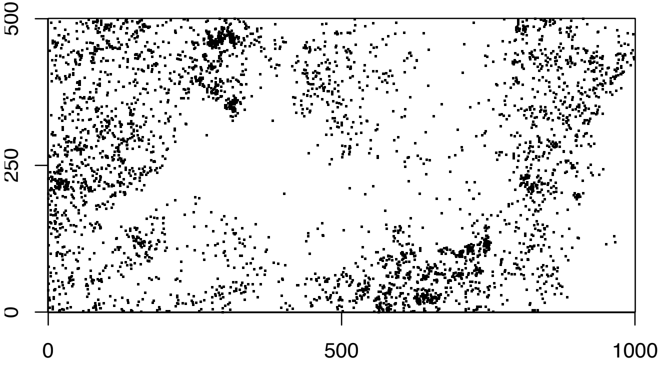
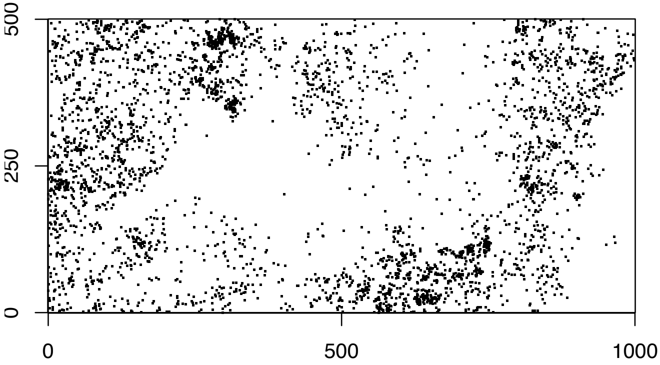


⇒ identifying and explaining structures in **point patterns**

spatial point processes



⇒ identifying and explaining structures in **point patterns**
stochastic models: **spatial point processes**

spatial point processes – what are they?

models of spatial patterns:

$\Rightarrow$ modelling **locations and properties ("marks")** of objects, events, individuals in space and time

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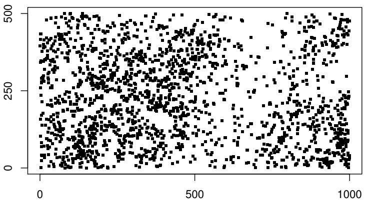

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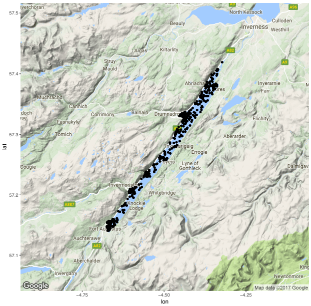

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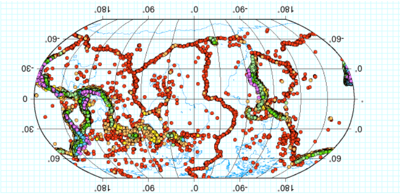

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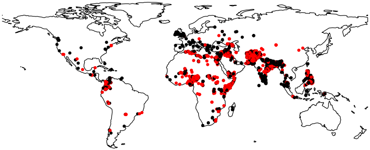

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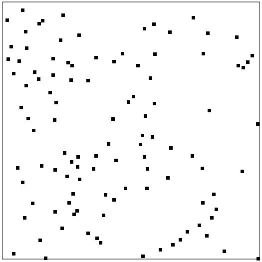

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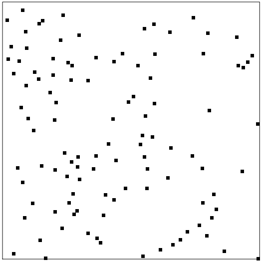

applications: medicine and health sciences, ecology, environmental sciences, international relations (terrorism studies), geology

Illian and Burslem, 2007 and 2017, Illian et al. 2008, Brown et al. 2011, 2013, 2016

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$\Rightarrow$ point process N is a random variable, whose values are measures

$\Rightarrow$ a **random (counting) measure**

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mathematically complex and intriguing

- how to translate statistical approaches into this context?

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- *mainly discussed in theoretical literature*

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- *simplifying assumptions: e.g. small patterns, in rectangular observation windows, rarely considering marks, every "point" has been seen and detected*

$\Rightarrow$ *models too far removed from reality*

$\Rightarrow$ *rarely used to answer scientific questions*

spatial point processes in ecology

in ecology

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⇒ **spatial point process methodology** should be highly relevant!

however...

- few ecologists aware of spatial point process methodology

⇒ not part of the standard statistical toolbox

spatial point processes in ecology

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WHY?

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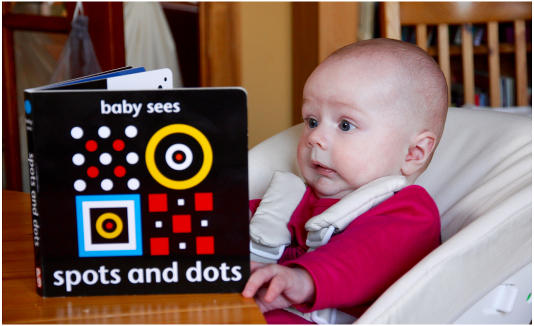
In the end it's just a bunch of dots, isn't it?



spatial point process modelling

aim

- *relevant and*
- *usable spatial point process methodology*

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approach: exploiting computational efficiency to construct realistically complex models – using INLA

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INLA (integrated nested Laplace approximation) is an alternative to MCMC

- much, much **faster**
- implemented in `R-INLA`
- suitable for a specific (but very large!) class of models

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$\Rightarrow$ very nice tool for Bayesian inference
$\Rightarrow$ computationally efficient model fitting, wide range of models

- quick
- accurate

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in essence:

$\Rightarrow$ computational efficiency and flexibility makes it realistic to fit complex models

point process modelling...

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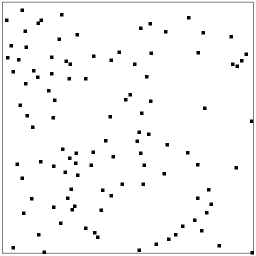

ignoring the real world – point process modelling

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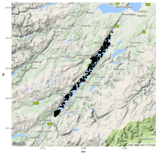

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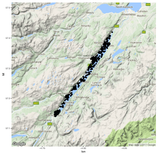

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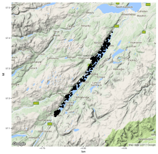

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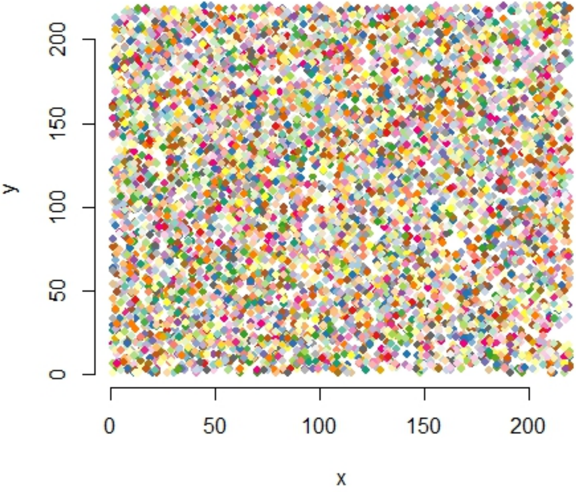
$\Rightarrow$ data have a complex observation process

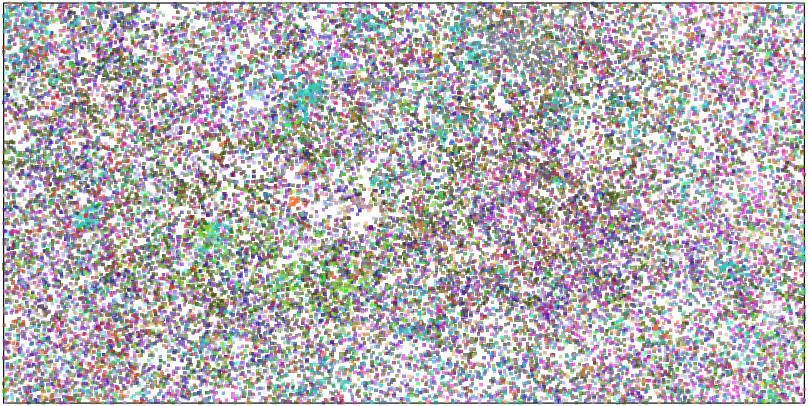
by contrast...

What I did 15 years ago:



and so we move on...

What we are discussing 15 years later:



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- flexibly jointly model marks and (spatial) covariates along with spatial pattern
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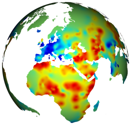
- model on complex domains
  - the sphere = the earth
  - observation areas with barriers (islands, archipelagos...)



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- we might have to think about **model assessment** for complex models... shudder....
- we need to speak to other scientists...

Examples...

aim

- *data collections issues – distance sampling*

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aim

- *data collections issues – distance sampling*
- *model interpretation – back to the rainforests*

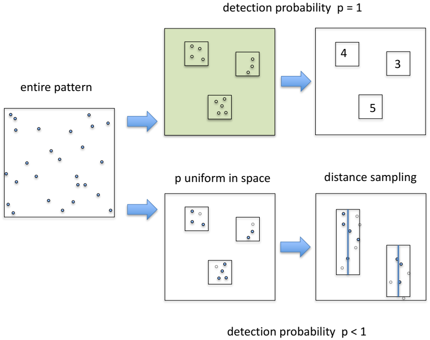
Examples...

aim

- *data collections issues – distance sampling*
- *model interpretation – back to the rainforests*
- *model assessment – discussion*

data collection...

In practice, especially in animal studies, observation area of interest is often too big to sample entirely.
thinned point process



data collection...

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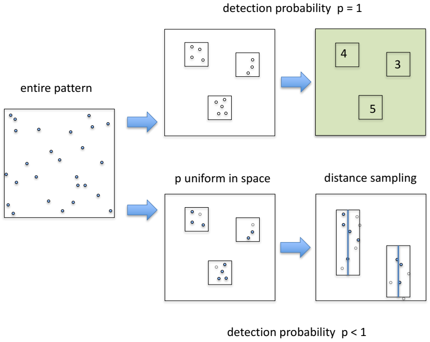

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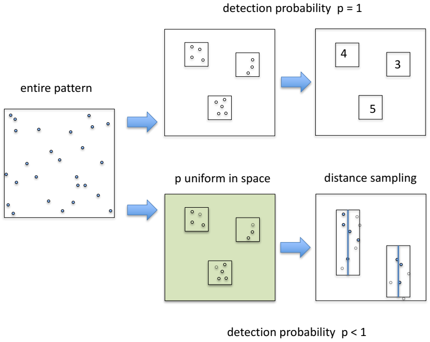

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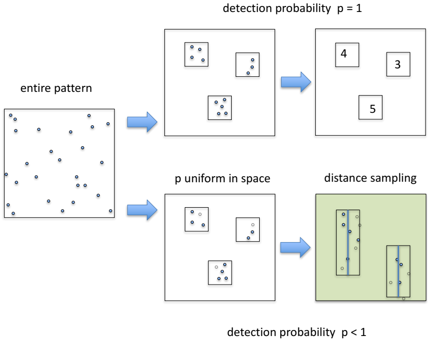

ecological surveys...

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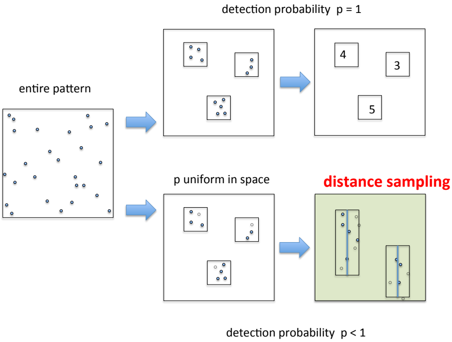
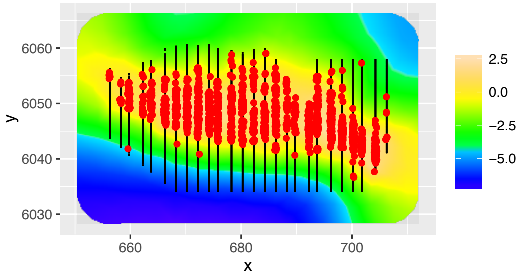


distance sampling data



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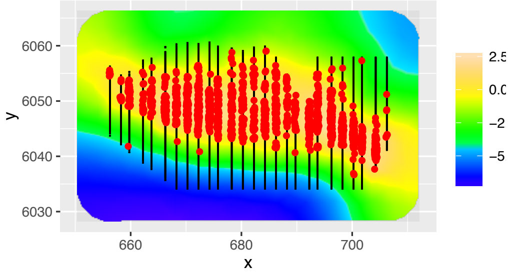

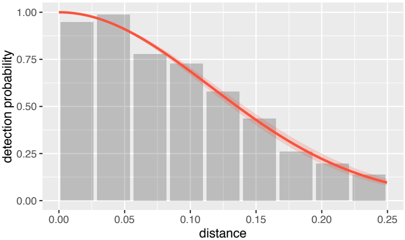

thinned point process!

example...

- large scale line-transect cetacean survey in the eastern tropical Pacific Ocean (ETP) between 1986 and 2007
- area of 21.353 million square kilometers ($>$ twice the size of Europe!) was surveyed (transects)
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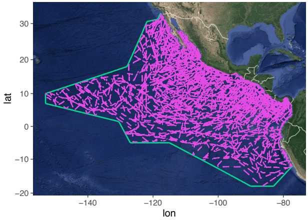
linear predictor depends on:

- detection function
- (SPDE-based) model for animal intensity
- integration scheme accounts for observation process



Yuan et al. 2016, Bachl et al. in preparation

distance sampling... nice...

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distance sampling... nice...

- spatio-temporal point process model
- models the effect of covariates continuously in space
- models spatial structure that cannot be explained by covariates
- elegant, integrated approach
- other data collection approaches may be seeing as **operations** on underlying point process
- implemented in *inlabru* (https://sites.google.com/inlabru.org/inlabru)

inference

the rainforests...

inference

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habitat association modelling:

inference

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- interest in understanding the mechanisms that allow different species to coexist

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- common approach: log-Gaussian Cox process

Model interpretation... and more

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spatially-unstructured random field $\boldsymbol{v} = \{v(s),\ s \in \Omega\}$: error field accounting for over-dispersion or clustering within grid cells

why...

Why do we need the spatial effects?

- model assumption: locations of individuals are independent – given the linear predictor
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- properties of the spatial field determine the smoothness of the spatial field
- choose well to avoid under- or overfitting!

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assign (typically Gamma) priors to the precision (inverse variance) parameters of the two random fields

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issues:

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3. scale spatially structured effect in terms of generalised variance

Sørbye et al. 2016, Sørbye et al. 2018

solutions...

make prior choice transparent and problem-driven

first step:

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now consider

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hyperparameters τ and ϕ: assign penalised complexity (PC) priors

Simpson et al., 2017

make prior choice transparent and problem-driven

second step:

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second step: re-think priors:

make prior choice transparent and problem-driven

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$\Rightarrow$ we don't know what the ideal smoothness/wigglyness is...

here: penalise something different – deviation from a base model

make prior choice transparent and problem-driven

penalise deviation from a base model...

Sequence of two base models here:

1. model with covariates, no random field

make prior choice transparent and problem-driven

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Sequence of two base models here:

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2. model with covariates and unstructured field, accounting for some local overdispersion

⇒ we have a parameter that reflects how close we are to each of the two base models

⇒ priors are chosen depending on how confident we are about this

inference – thoughts

common approach: log-Gaussian Cox process

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"we need to think about **model interpretation and inference** – BUT: we have very rarely answered users' scientific questions...

model assessment...

in practice:

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- difficult to use for point process models

model assessment for point processes

standard approach for fully observed point patterns:
envelop tests based on summary characteristics

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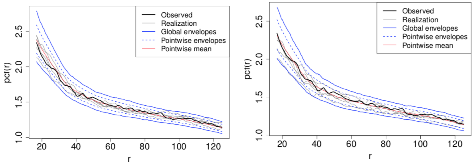

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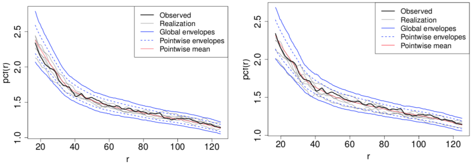

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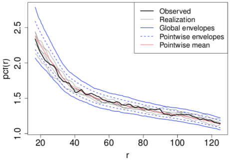

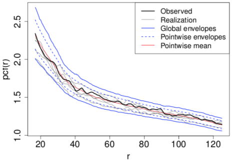

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- very little experience with comparison via standard approaches, i.e. AIC, DIC etc.

model assessment for point processes

Let's do some work

there is a lot to do here...

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- talk to other scientists: **communication of modelling process very important**
- listen to other scientists... : **methods for model assessment sorely needed**

recall those rainforests...

recall those rainforests...

aim

quantification of this

recall those rainforests...

aim

quantification of this

We can quantify the structure – but we still have very little to show in terms of answering practical questions...