

# INLA/SPDE WORKSHOP

Introduction of the air pollution dataset and elements of comparison between space-time estimation methods applied to air quality forecasting

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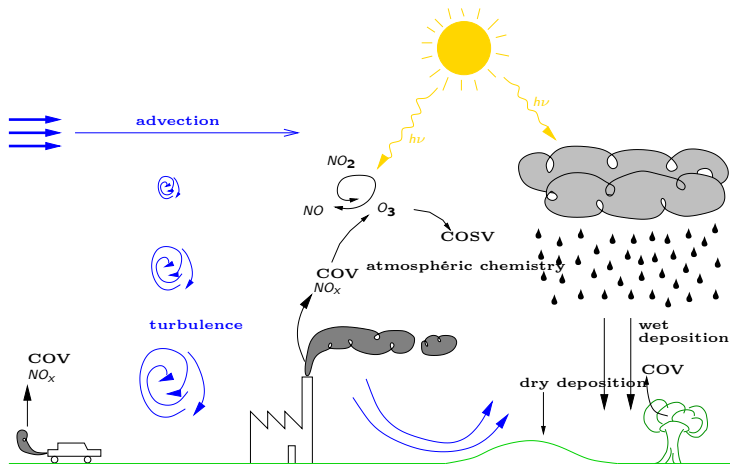
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November 8, 2018

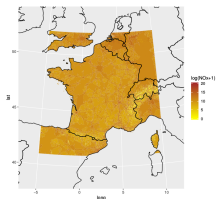
## Scientific context



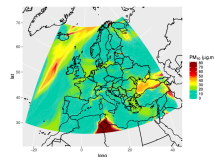
**Figure 1** – Atmospheric dynamics of the pollutants

## Scientific context

# Implementation of mathematical models to describe the evolution processes of the chemical species (pollutant) in the troposphere

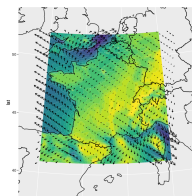


Emissions



Boundary conditions

CHIMERE



Meteo

## Air Pollution Dataset

### CHIMERE

The RObject CHIM\_2014.RData : data.frame with daily PM<sub>10</sub> and PM<sub>2.5</sub> CHIMERE simulations in 2014.

CHIMERE is run over the AFM French simulation domain, with a 10 km resolution

### Listing 1 – Reading the CHIMERE dataset

```

1 > load('CHIM_2014.RData')
2 > str(CHIM_daily2014)
3 'data.frame': 4092015 obs. of 5 variables:
4 $ long      : num  -5 -4.85 -4.7 -4.55 -4.4 ...
5 $ lat       : num  41 41 41 41 41 41 41 41 41 41 ...
6 $ date      : POSIXct, format: "2013-12-31" "2013-12-31" "2013-12-31" "2013-12-31" ...
7 $ PM10_CHIM: num   2.74 3.01 3.3 3.42 3.56 ...
8 $ PM25_CHIM: num   2.13 2.32 2.53 2.6 2.71 ...
9 > date = as.POSIXct("20140315",format="%Y%m%d",tz="UTC")
10 > simu = CHIM_daily2014[CHIM_daily2014$date==date,]
11 > str(simu)
12 'data.frame': 11211 obs. of 5 variables:
13 $ long      : num  -5 -4.85 -4.7 -4.55 -4.4 ...
14 $ lat       : num  41 41 41 41 41 41 41 41 41 ...
15 $ date      : POSIXct, format: "2014-03-15" "2014-03-15" "2014-03-15" "2014-03-15" ...
16 $ PM10_CHIM: num  12.4 12.9 13.4 13.6 13.7 ...
17 $ PM25_CHIM: num   9.47 9.98 10.42 10.6 10.57 ...

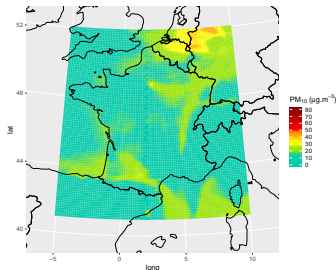
```

## Air Pollution Dataset

### CHIMERE

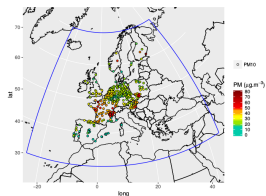
The RObject CHIM\_2014.RData : data.frame with daily mean of the PM<sub>10</sub> and PM<sub>2.5</sub> hourly CHIMERE simulations in 2014.

CHIMERE is run over the AFM french simulation domain, with a 10 km resolution



**Figure 2** – CHIMERE simulation (15<sup>th</sup> of March 2014)

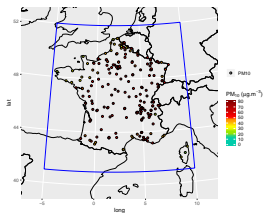
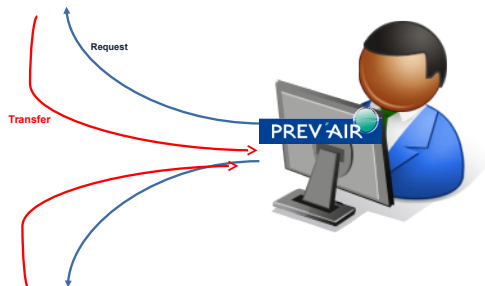
## Collecting the data



Europe

European database :

- 1) Ozoneweb (real-time data)
- 2) Airbase (validated data)



France

National database :

GEOD'AIR  
Real-time data  
Validated data

## Air Pollution Dataset

### Observations

The RObject `OBS_2014.Rdata` : `data.frame` with daily mean of the  $PM_{10}$  and  $PM_{2.5}$  hourly observations of the french GEOD'AIR database in 2014.

### Listing 2 – Reading the observational dataset

```

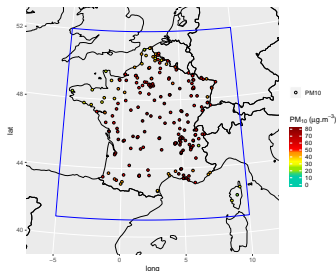
1 > load('OBS_2014.RData')
2 > str(OBS_daily2014)
3 'data.frame': 185055 obs. of 7 variables:
4 $ ID : Factor w/ 507 levels "FR01001","FR01005",...: 1 1 1 1 1 1 1 1 1 1 ...
5 $ long : num 5.8 5.8 5.8 5.8 5.8 ...
6 $ lat : num 49.5 49.5 49.5 49.5 49.5 ...
7 $ type_of_station: Factor w/ 5 levels "", "Background",...: 2 2 2 2 2 2 2 2 2 2 ...
8 $ date : POSIXct, format: "2014-01-01" "2014-01-02" "2014-01-03" "2014-01-04" ...
9 $ PM10 : num 6.8 4 10 10 7.9 5.8 8.7 8.8 6.1 9.2 ...
10 $ PM25 : num NA NA NA NA NA NA NA NA NA NA ...

```

## Air Pollution Dataset

### Observations

The RObject `OBS_2014.Rdata` : data.frame with daily mean of the  $PM_{10}$  and  $PM_{2.5}$  hourly observations of the french GEOD'AIR database in 2014.



**Figure 3** – Observations (15<sup>th</sup> of March 2014)



## PREV'AIR

Operational system for air quality monitoring and forecasting over Europe and France, under the aegis of the Ministry in charge of the environment

- ▶ Partners : INERIS, Météo-France, CNRS, IPSL, LCSQA
- ▶ Set up in 2003 to deliver daily AQ forecasts and maps on France & Europe
- ▶ Based on deterministic chemistry-transport modelling and post-processing using in situ observation data
- ▶ During pollution episodes, alert procedures are mainly triggered according to the forecast situation for the previous day (D-1) and next days (D+0, D+1, D+2)

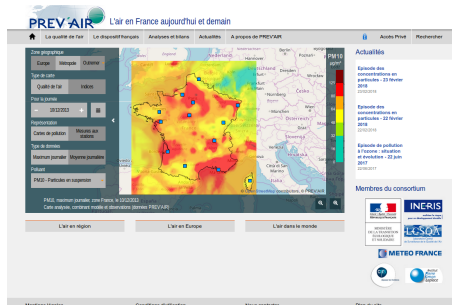


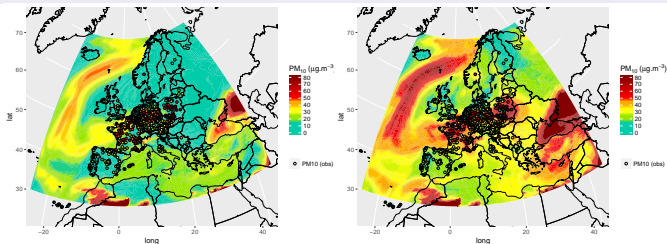
Figure 4 – Screenshot of the PREV'AIR website <http://www2.prevail.org/>

Two products are delivered by the PREV'Air system :

## 1) Analysis (Estimation problem)

Map of the previous day (D-1)

- 1) Meteorology, Emissions and Boundary conditions are used to run a CHIMERE simulation
- 2) Monitoring data are collected (France + Europe)
- 3) Kriging of **background** concentration measurements with CHIMERE data as external drift



(a) CHIMERE simulation

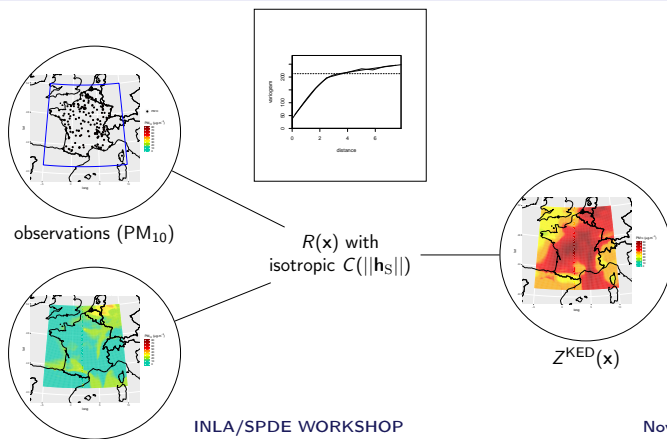
(b) Analysis (Kriging with external drift)

**Figure 5** – CHIMERE daily simulation and analysis (11<sup>th</sup> of March 2014)

## Kriging with external drift

In the kriging with external drift model (Chiles and Delfiner, 2012), the relation between the explanatory variables  $\varphi_I(\mathbf{x}_\alpha)$  (the model here) and the observations  $Z(\mathbf{x}_\alpha)$  is assumed to be linear :

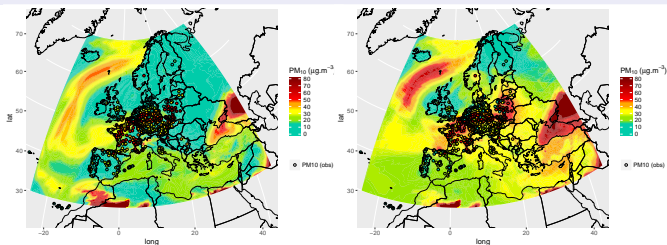
$$Z(\mathbf{x}) = \sum_I \beta_I \varphi_I(\mathbf{x}) + R(\mathbf{x})$$



## II) Forecast (Prediction problem)

Forecast maps of the days D+0, D+1, D+2

- 1) Meteorology, Emissions and Boundary conditions are used to run a CHIMERE simulation
- 2) Local forecasting at the background monitoring sites by Multilinear regressions (CI-TEAIRII project, 2011) or Generalized additive models (Lavancier, 2016; Valsania, 2016)
- 3) Kriging of **background** concentration measurements with CHIMERE data as external drift



(a) CHIMERE simulation

(b) Forecast (Kriging with external drift)

**Figure 3** – CHIMERE daily simulation and forecast (11<sup>th</sup> of March 2014)

## Advantages and Drawbacks

### Advantages

- ▶ Methodology implemented and evaluated for several years
- ▶ Improvement of the forecasts, especially for D+0

### Predictors at the monitoring stations :

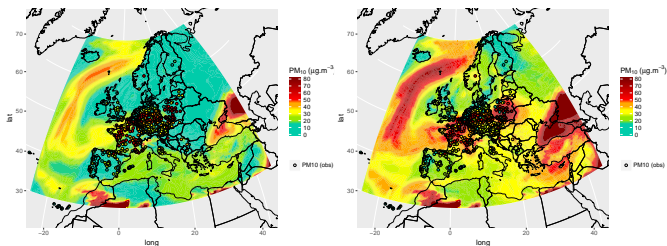
- 1) past observations (D-1 and first hours of D+0)
- 2) forecast meteorological variables
- 3) forecast concentrations

### Drawbacks

- ▶ The statistical models at the stations have to be trained again each time the CHIMERE model is upgraded
- ▶ New monitoring stations cannot be introduced in the forecast before one year (in order to have enough data for the training)

Run the CHIMERE and meteorological models is very costly

## The prediction problem



(a) CHIMERE simulation

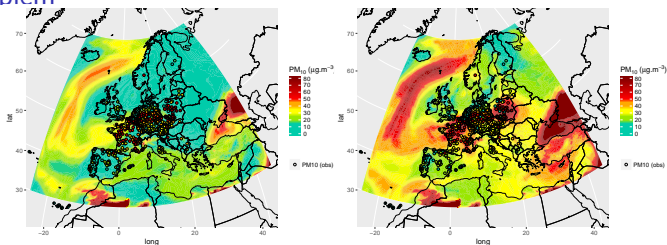
(b) Analysis (Kriging with external drift)

**Figure 4** – CHIMERE daily simulation and analysis (11<sup>th</sup> of March 2014)

The **prediction problem** is usually solved by DA techniques (see e.g. Asch et al., 2016)

In AQ,  $\text{impact}(\text{emissions}) > \text{impact}(\text{initial conditions}) \rightarrow$  space-time estimation techniques are very competitive (MACC project, 2015)

## The prediction problem



(a) CHIMERE simulation

(b) Analysis (Kriging with external drift)

**Figure 4** – CHIMERE daily simulation and analysis (11<sup>th</sup> of March 2014)

### Idea

Consider the analysis (D-1) and the statistical adaptation (D+0, D+1, D+2) as a single product in a spatio-temporal kriging framework

### Notations

Let  $Z(\mathbf{x}_\alpha, t_k)$ ,  $\alpha = 1, \dots, N$ ,  $k = 1, \dots, M - 1$  denote the space-time dataset of AQ concentrations observed at the monitoring sites  $\mathbf{x}_\alpha$  between time  $t_1$  and  $t_{M-1}$

In the 2016 RESSTE workshop and its related publication (Allard et al., 2017), the kriging of the daily  $PM_{10}$  bias of CHIMERE is used to produce the analysis :

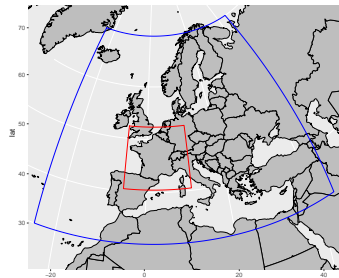
$$Z(\mathbf{x}, t) = \mu(\mathbf{x}, t) + R(\mathbf{x}, t) \quad (1)$$

with  $\mu(\mathbf{x}, t)$ , the local mean of the process is taken as the CHIMERE value and  $R(\mathbf{x}, t)$  is the residual, here the bias of the model

Based on this previous work, a large dataset is used to complete the kriging predictions with the PREV'AIR system predictions

## Data

- ▶ Type : Observations, CHIMERE and meteorological variables
- ▶ Pollutants :  $PM_{10}$  and  $O_3$
- ▶ Time resolution : daily
- ▶ Domains : Europe (2014) & France (2013)



**Figure 5** – MACC1e (blue) and FRA4k (red) domains



## The intercomparison exercise

### Operational context

- ▶ PREV'AIR has to provide the forecasts for D+0, D+1 and D+2 at D+0 09 :00
- ▶ Direct forecast of the daily mean concentration (**using only D-1 observations**)
- ▶ Daily mean concentration calculated as the average of the 24 hourly forecasts (**using D-1 observations & D+0 observations until 06 :00**)

Because big datasets are used, two options are used for the kriging :

- (1) Usual **covariance-based kriging** with CHIMERE as external drift (identified better than kriging the bias), with (small) space-time moving neighbourhood
- (2) **SPDE-based kriging** to deal with more data when solving the kriging system

## Outline

Presentation of the methods

Covariance-based kriging performance

Comparison with the statistical adaptation

Contributions of the SPDE-based kriging

## Statistical adaptation (PREV'AIR system)

step (1) : a Generalized Additive Model (**gam**) is built for each monitoring sites  $\mathbf{x}_\alpha$  :

$$Z(\mathbf{x}_\alpha, t_k) = \beta_0 + \sum_{i=1, \dots, p} f_i(\varphi_i(\mathbf{x}_\alpha, t_k)) + \varepsilon \quad (2)$$

where  $\varphi_i(.,.)$ ,  $i = 1, \dots, p$  are explanatory variables of the process  $Z(.,.)$ . The training dataset has to be long, several years if possible

step (2) : the estimation at location  $\mathbf{x}_0$  is given by a spatial kriging of the statistical forecasts obtained by these station-specific gam models

## Covariance-based kriging

$Z(\mathbf{x}, t)$  is a random function with deterministic part  $\mu(\mathbf{x}, t)$  and a residual  $R(\mathbf{x}, t)$  :

$$Z(\mathbf{x}, t) = \mu(\mathbf{x}, t) + R(\mathbf{x}, t) = \left[ \beta_0 + \sum_{i=1}^p \beta_i \varphi_i(\mathbf{x}, t) \right] + R(\mathbf{x}, t) \quad (3)$$

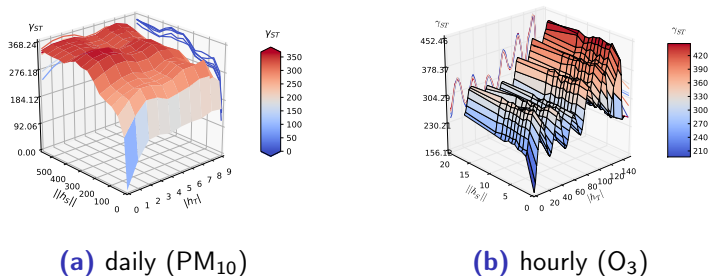
with the coefficients  $\beta_0$  and  $\beta_i$  unknown.

A space-time kriging  $Z(\mathbf{x}, t) = \sum_{\alpha, k} \lambda_{\alpha, k} Z(\mathbf{x}_\alpha, t_k)$  is used for the estimation.

The weights  $\lambda_{\alpha, k}$  are solution of the linear system (Chiles and Delfiner, 2012) :

$$\begin{cases} \sum_{\alpha=1}^n \lambda_\alpha \gamma(\mathbf{x}_\alpha - \mathbf{x}_\beta, t_k - t_l) + \mu_0 + \sum_{i=1}^p \mu_i \varphi_i(\mathbf{x}_\beta, t_l) & = \gamma(\mathbf{x}_\beta - \mathbf{x}_0, t_k - t_0) \quad \forall \beta \\ \sum_{\alpha=1}^n \lambda_\alpha & = 1 \\ \sum_{\alpha=1}^n \lambda_\alpha \varphi_i(\mathbf{x}_\alpha, t_k) & = \varphi_i(\mathbf{x}_0, t_0) \quad \forall i \end{cases} \quad (4)$$

where  $\gamma(\cdot, \cdot)$  denotes a space-time authorized variogram model, (see e.g. Gneiting et al., 2007; Porcu et al., 2006; De Iaco et al., 2001)



**Figure 6** – Examples of daily and hourly variograms

## Advantages

- ▶ Space-time moving neighbourhood  $\rightarrow$  local fitting of  $\beta_i$

## Drawbacks

- ▶ The neighbourhood has to be small for reasonable inversion CPU time
- ▶ Small neighbourhood  $\rightarrow$  using meteorological variables as predictors  $\varphi_i$  is useless (no variability)

## SPDE-based kriging I

Model (Cameletti et al., 2012) :

$$Z(\mathbf{x}, t) = \beta_0 + \underbrace{\sum \beta_i \varphi_i(\mathbf{x}, t)}_{\text{local mean}} + \underbrace{\xi(\mathbf{x}, t)}_{\text{latent field}} + \underbrace{\varepsilon(\mathbf{x}, t)}_{\text{obs error}} \quad (5)$$

with  $\varepsilon(\mathbf{x}, t) \sim \mathcal{N}(0, \sigma_\varepsilon^2)$  and the latent field is an AR1 process :

$$\xi(\mathbf{x}, t) = a\xi(\mathbf{x}, t-1) + \omega(\mathbf{x}, t) \quad (6)$$

with  $\omega(\mathbf{x}, t) \sim \mathcal{N}(0, \sigma_\omega^2 C(h))$ ,  $C(\mathbf{h})$  a Matérn (spatial) covariance.

### Coupled SPDE/INLA approach

- (1) Rewrite the Model (5) based on the SPDE representation of the Gaussian field
- (2) Estimation of the parameters in Model (5) with INLA, see e.g. Opitz (2016)

## SPDE-based kriging II

A separable space-time covariance is built by approximating the Gaussian field by its Finite Elements representation :

$$\xi(\mathbf{x}, t) = \sum_k \psi_l(\mathbf{x}, t) \omega_k = \sum_k \psi_i^s(\mathbf{x}) \psi_j^t(t) \omega_k \quad (7)$$

where the basis functions are seen as the product of purely spatial basis functions  $\psi_i^s(\mathbf{s})$  and purely temporal basis functions  $\psi_j^t(t)$ , then the space-time stochastic PDE (Lindgren et al., 2011) defined by :

$$\frac{\partial}{\partial t} (\kappa(\mathbf{x})^2 - \Delta)^{\alpha/2} (\tau(\mathbf{x}) \xi(\mathbf{x}, t)) = \mathcal{W}(\mathbf{x}, t), \quad (\mathbf{x}, t) \in \mathcal{D} \times \mathbb{R}$$

generates a precision matrix  $\mathbf{Q}$  for the Gaussian weights  $\omega_k$  so that :

$$\mathbf{Q} = \mathbf{Q}_T \otimes \mathbf{Q}_S$$

$\mathbf{Q}_S$  and  $\mathbf{Q}_T$  are respectively the precision matrices of the purely spatial model and the Markovian random walk.

## Outline

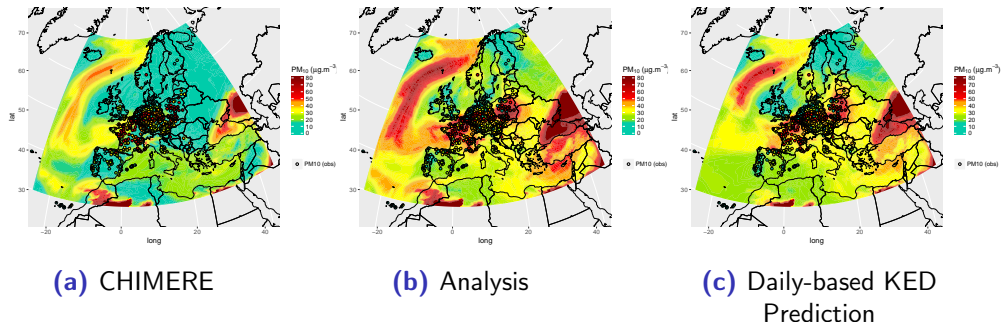
Presentation of the methods

Covariance-based kriging performance

Comparison with the statistical adaptation

Contributions of the SPDE-based kriging





**Figure 7** – CHIMERE, analysis and daily KED predictions (11<sup>th</sup> of March 2014)

## Direct kriging forecast of the daily mean concentration

- ▶ Most of the Western Europe patterns in the analysis are in the forecast...
- ▶ But still some strong differences
- ▶ Big differences in far-off spatial extrapolations

## (I) CHIMERE

$C(\mathbf{x}_\beta, t_0)$ , the daily outputs of CHIMERE interpolated at location  $(\mathbf{x}_\beta, t_0)$

## (II) ANALYSIS (LOOCV)

To estimate  $Z(\mathbf{x}_\beta, t_0)$ , the dataset is  $\{Z(\mathbf{x}_\alpha, t_k)\}$ ,  $(\mathbf{x}_\alpha, t_k) \neq (\mathbf{x}_\beta, t_0)$  is used

How the spatial information brought by the neighbours at  $D+0$  helps for the estimation of  $Z(\mathbf{x}, t)$  at a location known in the past but not at the current time

## (III) FORECAST (LOOCV)

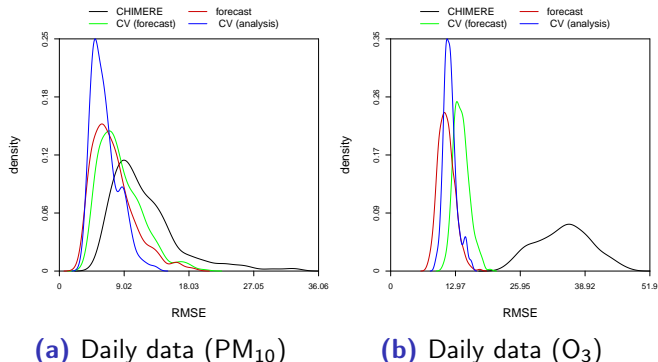
To estimate  $Z(\mathbf{x}_\beta, t_0)$ , the dataset is  $\{Z(\mathbf{x}_\alpha, t_k)\}$ ,  $\alpha \neq \beta$ ,  $k \neq 0$  is used : the time series in  $(\mathbf{x}_\alpha, t_k)$  is removed

Assess the performance of the prediction without any information in space or time

## (IV) FORECAST

To estimate  $Z(\mathbf{x}_\beta, t_0)$ , the dataset is  $\{Z(\mathbf{x}_\alpha, t_k)\}$ ,  $k \neq 0$  is used

The operational score



**Figure 9** – RMSE

**Logical order of performance :**  
 analysis > forecast > forecast in cross-validation > CHIMERE

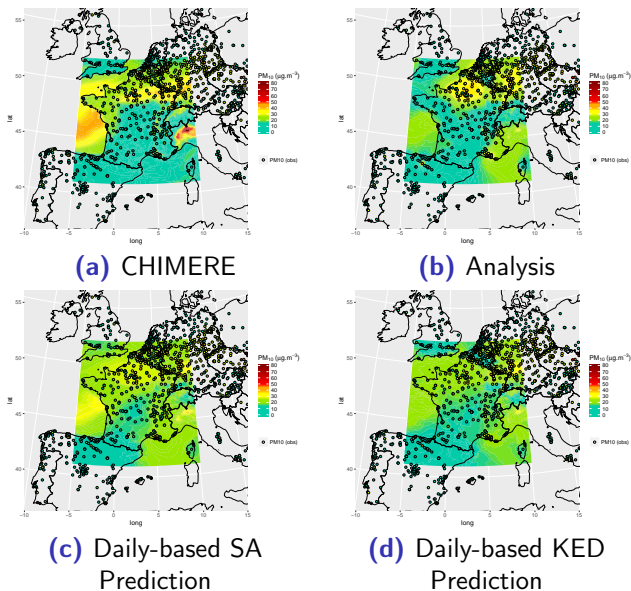
## Outline

Presentation of the methods

Covariance-based kriging performance

Comparison with the statistical adaptation

Contributions of the SPDE-based kriging



**Figure 10** – CHIMERE, analysis and daily-based GAM & KED predictions (11<sup>th</sup> of March 2013)

## Direct Forecast of the daily mean concentration

- Pollution plume over the North of France is better predicted by the statistical adaptation
- Maps very similar (consistent with the scores)

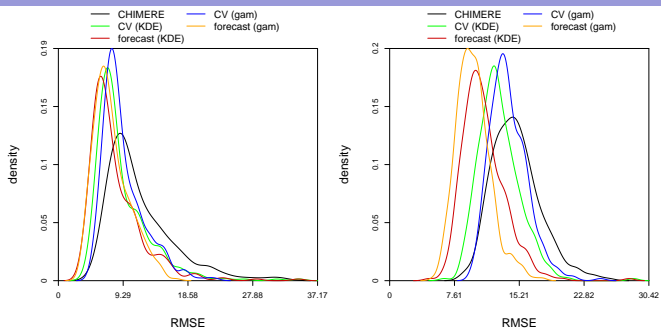
(a) Daily data (PM<sub>10</sub>)(b) Daily data (O<sub>3</sub>)

Figure 13 – RMSE

**Forecast :**

Performance very similar for PM<sub>10</sub>

Statistical adaptation better for O<sub>3</sub>...

**Cross-validation :**

Close to the monitoring sites, SA is better

Elsewhere, the kriging is competitive

## Outline

Presentation of the methods

Covariance-based kriging performance

Comparison with the statistical adaptation

Contributions of the SPDE-based kriging

## Remark

- ▶ Forecast scores only
- ▶ Reduced dataset (second semester of 2013 for  $PM_{10}$ , second quarter of 2013 for  $O_3$ )

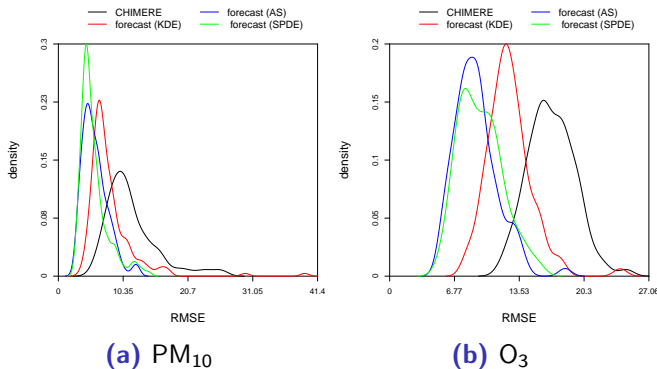


Figure 15 – RMSE

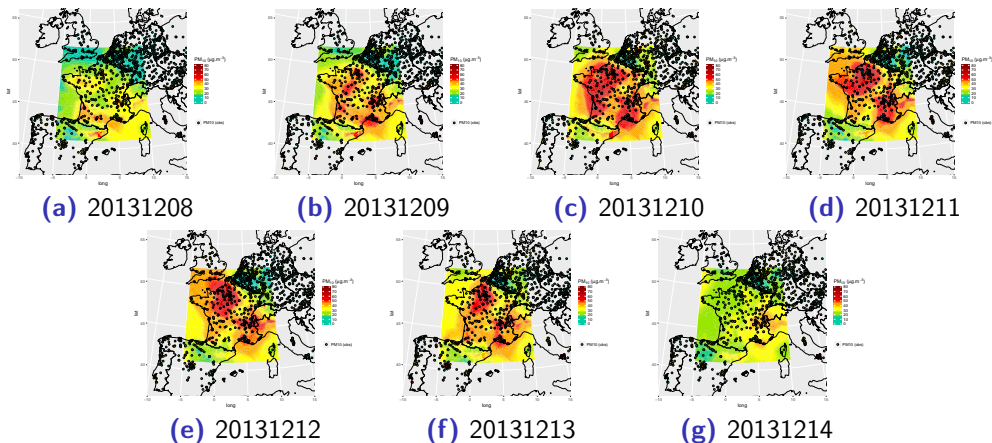
SPDE-based kriging better for  $PM_{10}$   
 Statistical adaptation better for  $O_3$  but SPDE-based kriging competitive



## The predictive skills of the SPDE-based kriging approach is an important result

### Case study

Pollution episode of December 2013, starting from the 9<sup>th</sup> and ending on the 14<sup>th</sup>



**Figure 16** – Daily analyses during the pollution episode of December 2013

## SPDE-based kriging better for the beginning and the end of pollution episodes

		Normalized Mean Bias		
		KED	GAM	SPDE
Day	20131207	17,26	2,76	-5,51
	20131208	12,46	4,32	-6,88
	20131209	-15,87	-12,98	-10,43
	20131210	-31,07	-14,61	-7,17
	20131211	-1,92	-8,57	-11,63
	20131212	-2,74	-12,65	-11,97
	20131213	-2,21	-8,47	-9,91
	20131214	21,36	9,24	-3,33

**Table 1** – Normalized Mean Bias during the pollution episode of 2013

### Why ?

Statistical adaptation uses the (D-1) observation as a predictor for the local mean  $\mu(\mathbf{x}, t)$

Usual kriging approach : few data, only CHIMERE as covariate  $\rightarrow$  poorly estimates the drift

SPDE/INLA approach : more data, more explanatory variables  $\rightarrow \text{Var}[R(\mathbf{x}, t)] \searrow$

## Conclusion

The main questions were :

1) **How does spatio-temporal kriging compare to the approach used in PREV' AIR to adjust CHIMERE forecasts ?**

Well. In addition, cross-validation results suggest good performance of spatio-temporal kriging in areas with sparse monitoring network.

2) **Does the coupled INLA-SPDE-based kriging approach bring any additional contribution to the performance of the usual covariance-based kriging ?**

Yes. Thanks to a bigger amount of data and explanatory variables, the INLA approach provides a better estimation of the local mean, which is a key point for the prediction of AQ pollution episodes.

## Acknowledgements

This study was funded by the French Ministry in charge of Environment

**Thank you for your attention**

## References I

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