

A poorman's spatio-temporal gridded HBM to handle 0-inflated biomass data

Jean-Baptiste Lecomte, Éric Parent, Liliane Bel, Marie P. Etienne

AgroParisTech

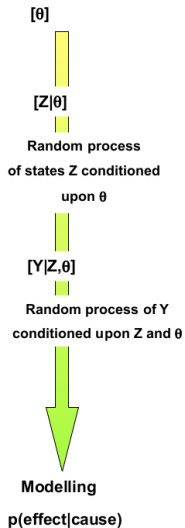
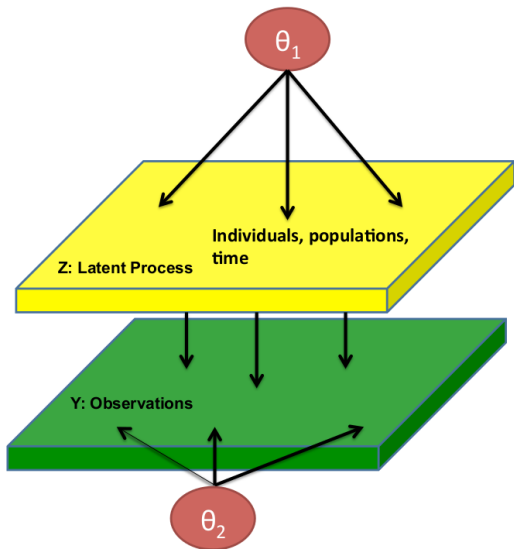
Contributor: Hugues Benoit(Environnement Canada, Moncton)

Mai 16, 2017

- 1 No longer affordable Hierarchical Spatio-Temporal Bayesian model ?
- 2 A zero-inflated Spatio-temporal HBM
- 3 Results
- 4 Conclusion & perspectives

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Observation/Latent/Parameters



An up-to-date methodological impulse towards highly sophisticated spatio-temporal frameworks

The sandwich layer has grown uneatable

Are things getting nowadays overbig?

- sample size grows: Records of environmental data to be indexed with s, t
- model sophistication increases : Environmental data more alike when closer in time and space.

HBM as an attempt to distangle complexity:

$$\theta \sim \pi(\theta) \text{ parameters}$$

$$Z|\theta \sim g(z|\theta) \text{ latent var.}$$

$$Y|z, \theta \sim \prod_i^n f(y_i|z_i, \theta) \text{ data}$$

Model and Inference challenges although workable multivariate process model: $g(\cdot)$ often normal with $\mathbb{E}(Z|\theta) = \mu_\theta(\cdot), \mathbb{V}(Z|\theta) = \Sigma_\theta(\cdot, \cdot)$

Some good and bad reasons to keep it simple

Making compromises is unavoidable

Two possible strategies:

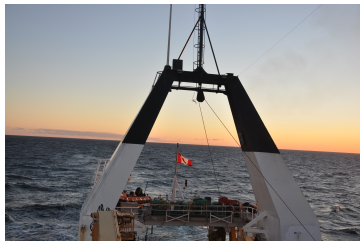
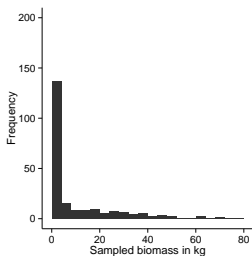
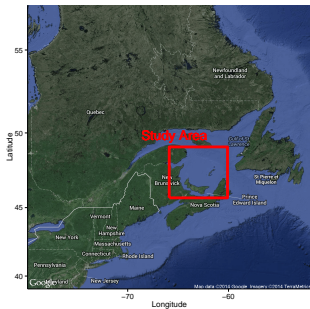
- 1 Simplify the model:
 - approximate with a Gaussian Markov Random Field (Lindgren et al., 2011) requiring $O(n \log n)$ operations
 - work out low rank methods (Cressie and Johannesson, 2008) $O(n)$
- 2 Keep the model but forget about the "golden standards" for inference:
 - Frequentists: pseudo, partial, composite, (...) -likelihoods e.g. Curriero and Lele (1999)
 - Bayesians: INLA, Variational Bayes, etc.

Basic requirements

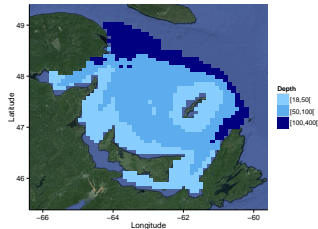
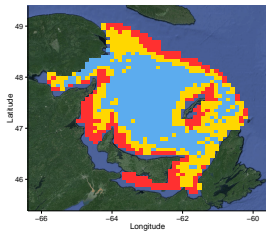
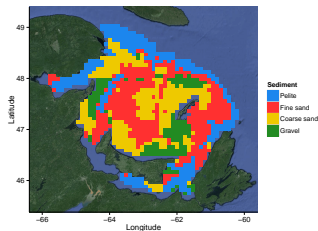
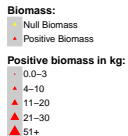
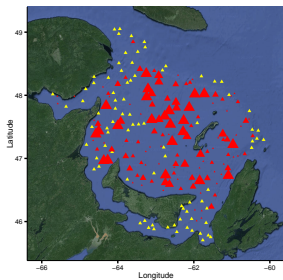
A minima

- Favoring a straight interface with others, possible non aware of statistical modeling
- Allowing for some increase of the database,
- Observation submodel may be specific,
- Trade-off between biologic realism and model complexity, isn't the best inference procedure the one you know?

Urchins in the south Saint-Lawrence Gulf



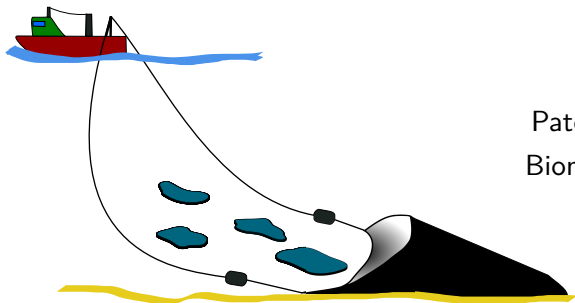
Urchins in the south Saint-Lawrence Gulf (1997)



Spatio-temporal data

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Observation process: A compound Poisson Gamma model



s site, $i \in s$

Patches: $N_s \sim \text{Poisson}(\lambda_s)$

Biomass: $M_{s,i} \sim \text{Gamma}(a_s, b_s)$

$$Y_s = \sum_{i=1}^{N_s} M_{s,i}$$

(λ_s, a_s, b_s) are latent variables stemming from the (spatio-temporal) process layer.

Y_s takes 0 when $N_s = 0$

Tweedie model not in INLA

Process Convolution: from Random fields to Mixed effects

If $X(s)$ continuous white noise over domain S , $k(\cdot)$ smoothing kernel:

$$Z(s) = \int_{u \in S} X(u) k(u - s) du$$

$$\Delta = s - s'$$

$$c(d) = \text{cov}(Z(s), Z(s')) = \int_{u \in S} k(u) k(u - d) du$$

Z isotropic : The spectrum of $c(\cdot)$ is the square of the Fourier transform of $k(\cdot)$

Space-time extension

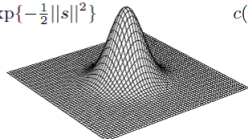
$$Z(s, t) = \int_{u \in S} X(u, t) k(u - s) du$$

Process Convolution: from Random fields to Mixed effects

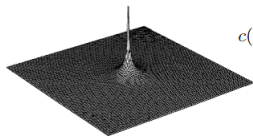
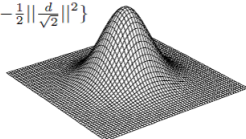
kernel

covariance function

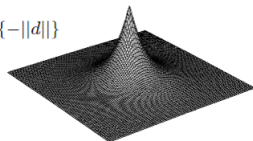
$$k(s) \propto \exp\{-\frac{1}{2}\|s\|^2\}$$



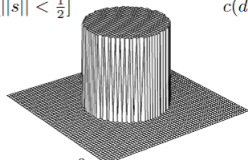
$$c(d) \propto \exp\{-\frac{1}{2}\|\frac{d}{\sqrt{2}}\|^2\}$$



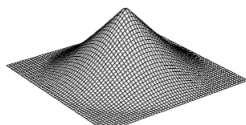
$$c(d) \propto \exp\{-\|d\|\}$$



$$k(s) \propto I[\|s\| < \frac{1}{2}]$$



$$c(d) \propto (1 - \frac{3}{2}\|d\| + \frac{1}{2}\|d\|^3)I[d < 1]$$



Process Convolution: from Random fields to Mixed effects

Process convolution approach for space+AR for time
 (patch nb $\lambda(s, t) \leftarrow Z(s, t) + \text{Covariates}$)

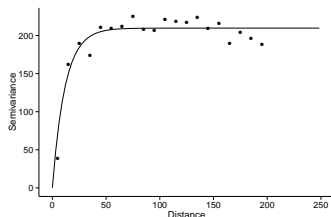
$$Z(s, t) \approx \sum_{g=1}^G X_g(t) K(g, s)$$

$$X(t+1) = \rho X(t) + \varepsilon(t)$$

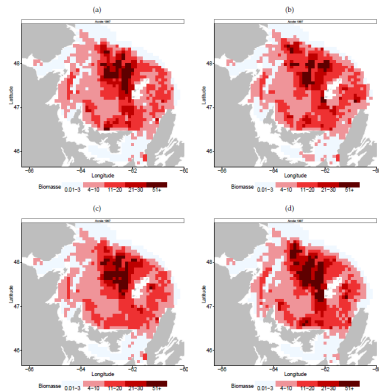
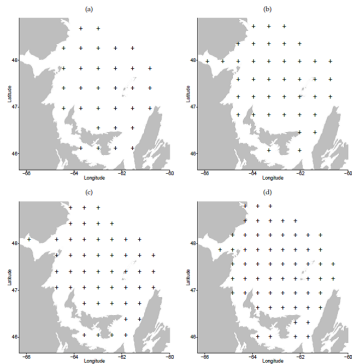
$$\varepsilon \sim N(0, \sigma^2 I)$$

$$K(g, s) = \exp - \left(\frac{d(s, g)}{\phi} \right)$$

(G grid points)



Does the grid size matter?



Nb points	32	41	50	63
DIC	1155.7	1146.2	1137.8	1175.7

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Inference

- Mixed-effect hierarchical model,
- Ready-made framework,
- Launch compiled BUGS code from NIMBLE.

Friendly softwares, e.g. WinBUGS®



"Bayesian Inference Using Gibbs Sampler"

Medical Research Council, Biostatistics Unit, Cambridge, UK

Imperial College, London, UK

- Bayesian statistical modeling
- Very popular
- Sampling based methods using Gibbs sampling (eventually hybrid)
- Models can be described graphically
- Freeware, available at : www.mrc-bsu.cam.ac.uk/bugs/
- Extensions : { Spatial models : GeoBUGS®
Convergence : Coda®, Boa® (R packages)
- Coupling with R

Mean posterior presence probability

Mean posterior expected biomass

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Conclusion

- Low cost spatio-temporal approach,
- Misalignment is no longer an issue,
- Model is not anymore curved by the sample locations,
- Time and space are modeled one after another.

Perspectives

- Computational burden : 3 hours!
- Model with more realistic ecological transition,
- Selection of explanatory variables,
- Irregular grids, special grid location for zeros,
- Consider the grid center locations as an unknown of the model,
- Pushing to higher dimensions (Multivariate extensions for Spatio-temporal Predator/prey models).

Thanks for your attention !

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