Estimation par maximum de vraisemblance par paires de champs gaussiens multivariés spatio-temporels

Marc Bourotte[1], Denis Allard[1] & Emilio Porcu[2]

[1] INRA Avignon - Unité BioSP
[2] University Federico Santa Maria, Chile

11 Mai 2015 : Journée du réseau RESSTE

Outline

Outline

Context

- Use of Gaussian Random Field **Z** (for example to model climatic variables)
- Multivariate space-time data

Second-order stationarity

For all $(\mathbf{s},t),(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$\begin{aligned} \mathbb{E}[Z_i(\mathbf{s},t)] &= m_i \\ \mathbb{C}ov\left[Z_i(\mathbf{s},t), Z_j(\mathbf{s}+\mathbf{h},t+u)\right] &= \mathbb{E}\left[(Z_i(\mathbf{s},t)-m_i).(Z_j(\mathbf{s}+\mathbf{h},t+u)-m_j)\right] \\ &= C_{ij}(\mathbf{h},u) \end{aligned}$$

Need to model a multivariate spatio-temporal second order structure

Separability

For all $(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$C_{ij}(\mathbf{h},u) = \rho_{ij}.C_S(\mathbf{h}).C_T(u) \qquad M \bigotimes S \bigotimes T$$

Context

- Use of Gaussian Random Field **Z** (for example to model climatic variables)
- Multivariate space-time data

Second-order stationarity

For all $(\mathbf{s},t),(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$\begin{aligned} \mathbb{E}[Z_i(\mathbf{s},t)] &= m_i \\ \mathbb{C}ov\left[Z_i(\mathbf{s},t), Z_j(\mathbf{s}+\mathbf{h},t+u)\right] &= \mathbb{E}\left[(Z_i(\mathbf{s},t)-m_i).\left(Z_j(\mathbf{s}+\mathbf{h},t+u)-m_j\right)\right] \\ &= C_{ij}(\mathbf{h},u) \end{aligned}$$

Need to model a multivariate spatio-temporal second order structure

Separability

For all $(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$C_{ij}(\mathbf{h},u) = \rho_{ij}.C_S(\mathbf{h}).C_T(u) \qquad M \bigotimes S \bigotimes T$$

Context

- Use of Gaussian Random Field **Z** (for example to model climatic variables)
- Multivariate space-time data

Second-order stationarity

For all $(\mathbf{s},t),(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$\begin{aligned} \mathbb{E}[Z_i(\mathbf{s},t)] &= m_i \\ \mathbb{C}ov\left[Z_i(\mathbf{s},t), Z_j(\mathbf{s}+\mathbf{h},t+u)\right] &= \mathbb{E}\left[(Z_i(\mathbf{s},t)-m_i).\left(Z_j(\mathbf{s}+\mathbf{h},t+u)-m_j\right)\right] \\ &= C_{ij}(\mathbf{h},u) \end{aligned}$$

Need to model a multivariate spatio-temporal second order structure

Separability

For all $(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$C_{ij}(\mathbf{h},u) = \rho_{ij}.C_S(\mathbf{h}).C_T(u) \qquad M \bigotimes S \bigotimes T$$

Context

- Use of Gaussian Random Field **Z** (for example to model climatic variables)
- Multivariate space-time data

Second-order stationarity

For all $(\mathbf{s},t),(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$\begin{aligned} \mathbb{E}[Z_i(\mathbf{s},t)] &= m_i \\ \mathbb{C}ov\left[Z_i(\mathbf{s},t), Z_j(\mathbf{s}+\mathbf{h},t+u)\right] &= \mathbb{E}\left[(Z_i(\mathbf{s},t)-m_i).\left(Z_j(\mathbf{s}+\mathbf{h},t+u)-m_j\right)\right] \\ &= C_{ij}(\mathbf{h},u) \end{aligned}$$

Need to model a multivariate spatio-temporal second order structure

Separability

For all $(\mathbf{h},u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i,j = 1,...,p$

$$C_{ij}(\mathbf{h},u) = \rho_{ij}.C_S(\mathbf{h}).C_T(u) \qquad M \bigotimes S \bigotimes T$$

Context

Nonseparable models :

- $M \bigotimes (ST)$: $C_{ij}(\mathbf{h}, u) = \rho_{ij}.C(\mathbf{h}, u) \rightarrow$ *Gneiting (2002)*
- $(MS) \bigotimes T$: $C_{ij}(\boldsymbol{h}, u) = C_{ij}(\boldsymbol{h}).C_T(u) \rightarrow$ *Gneiting, Kleiber & Schlather (2010) and Apanasovich, Genton & Sun (2012)*

Matérn covariance function :

$$M(\mathbf{h}|\nu, r) = \frac{2^{1-\nu}}{\Gamma(\nu)} (r\|\mathbf{h}\|)^{\nu} \mathscr{K}_{\nu}(r\|\mathbf{h}\|), \quad \mathbf{h} \in \mathbb{R}^d$$

Model

For all $(\mathbf{h}, u)$ in $\mathbb{R}^d \times \mathbb{R}$ and all $i, j = 1, ..., p$

$$C_{ij}(\mathbf{h}, u) = \frac{\sigma_i \sigma_j \beta_{ij}}{a|u|^{2\alpha}+1} \frac{\Gamma\left\{(\nu_i+\nu_j)/2\right\}}{\Gamma(\nu_i)^{1/2}\Gamma(\nu_j)^{1/2}} \frac{r_i^{\nu_i} r_j^{\nu_j}}{\{(r_i^2+r_j^2)/2\}^{(\nu_i+\nu_j)/2}}$$

$$\times \; M\left(\frac{\mathbf{h}}{(a|u|^{2\alpha}+1)^{\beta/2}} | \frac{\nu_i+\nu_j}{2}, \sqrt{\frac{r_i^2+r_j^2}{2}}\right) \qquad MST$$

Context

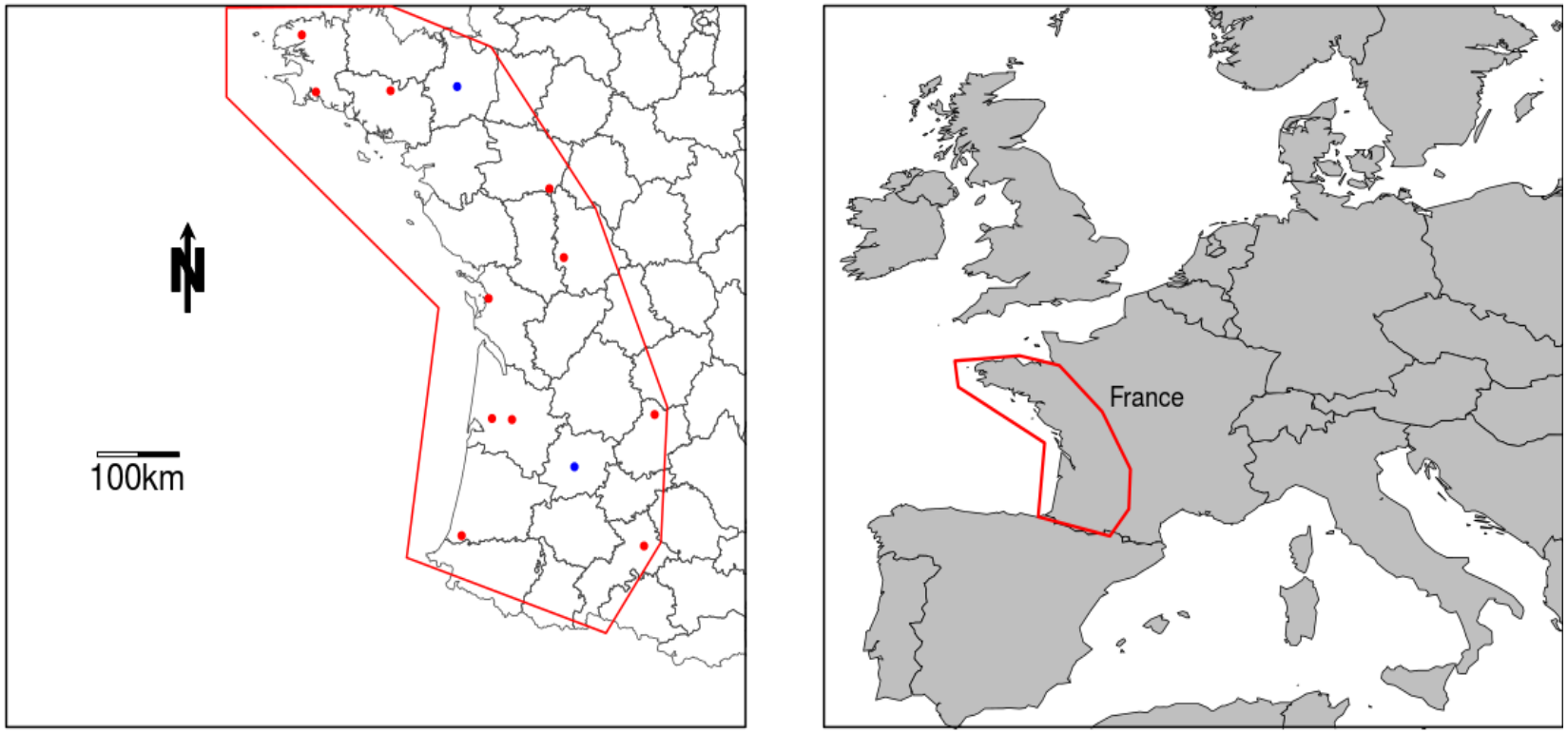

Figure : *Location of the 13 weather stations over western France. Red points are used for estimation, blue points for validation (INRA Climatik portal).*

Outline

Sub-likelihood

- Pairwise marginal likelihood $\left(Z_i(\mathbf{s}_\alpha, t_\alpha), Z_j(\mathbf{s}_\beta, t_\beta)\right)^\mathsf{T}$
- Pairwise conditional likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) | Z_j(\mathbf{s}_\beta, t_\beta)$
- Pairwise difference likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) - Z_j(\mathbf{s}_\beta, t_\beta)$

$$\begin{pmatrix} Z_i(\mathbf{s}_\alpha, t_\alpha) \\ Z_j(\mathbf{s}_\beta, t_\beta) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_i^2 & C_{ij}(\mathbf{h}, u) \\ C_{ij}(\mathbf{h}, u) & \sigma_j^2 \end{pmatrix} \right)$$

Sub-likelihood

$$l(i, j, \mathbf{s}_\alpha, \mathbf{s}_\beta, t_\alpha, t_\beta; \theta) = \frac{1}{2} \left\{ \log \Delta_{ij,\alpha\beta} + \frac{A_{ij,\alpha\beta}}{\Delta_{ij,\alpha\beta}} \right\}$$

where $\mathbf{h} = \|\mathbf{s}_\alpha - \mathbf{s}_\beta\|, u = |t_\alpha - t_\beta|, \Delta_{ij,\alpha\beta} = \sigma_i^2 \sigma_j^2 - C_{ij}(\mathbf{h}, u)^2$ and $A_{ij,\alpha\beta} = \sigma_j^2 Z_i(\mathbf{s}_\alpha, t_\alpha)^2 - 2 C_{ij}(\mathbf{h}, u) Z_i(\mathbf{s}_\alpha, t_\alpha) Z_j(\mathbf{s}_\beta, t_\beta) + \sigma_i^2 Z_j(\mathbf{s}_\beta, t_\beta)^2$.

Sub-likelihood

- Pairwise marginal likelihood $\left(Z_i(\mathbf{s}_\alpha, t_\alpha), Z_j(\mathbf{s}_\beta, t_\beta)\right)^\mathsf{T}$
- Pairwise conditional likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) | Z_j(\mathbf{s}_\beta, t_\beta)$
- Pairwise difference likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) - Z_j(\mathbf{s}_\beta, t_\beta)$

$$\begin{pmatrix} Z_i(\mathbf{s}_\alpha, t_\alpha) \\ Z_j(\mathbf{s}_\beta, t_\beta) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_i^2 & Cij(\mathbf{h}, u) \\ Cij(\mathbf{h}, u) & \sigma_j^2 \end{pmatrix} \right)$$

Sub-likelihood

$$l(i, j, \mathbf{s}_\alpha, \mathbf{s}_\beta, t_\alpha, t_\beta; \theta) = \frac{1}{2} \left\{ \log \Delta_{ij,\alpha\beta} + \frac{A_{ij,\alpha\beta}}{\Delta_{ij,\alpha\beta}} \right\}$$

where $\mathbf{h} = \|\mathbf{s}_\alpha - \mathbf{s}_\beta\|, u = |t_\alpha - t_\beta|, \Delta_{ij,\alpha\beta} = \sigma_i^2 \sigma_j^2 - C_{ij}(\mathbf{h}, u)^2$ and $A_{ij,\alpha\beta} = \sigma_j^2 Z_i(\mathbf{s}_\alpha, t_\alpha)^2 - 2\, C_{ij}(\mathbf{h}, u) Z_i(\mathbf{s}_\alpha, t_\alpha) Z_j(\mathbf{s}_\beta, t_\beta) + \sigma_i^2 Z_j(\mathbf{s}_\beta, t_\beta)^2$.

Sub-likelihood

- Pairwise marginal likelihood $\left(Z_i(\mathbf{s}_\alpha, t_\alpha), Z_j(\mathbf{s}_\beta, t_\beta)\right)^\mathsf{T}$
- Pairwise conditional likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) | Z_j(\mathbf{s}_\beta, t_\beta)$
- Pairwise difference likelihood $Z_i(\mathbf{s}_\alpha, t_\alpha) - Z_j(\mathbf{s}_\beta, t_\beta)$

$$\begin{pmatrix} Z_i(\mathbf{s}_\alpha, t_\alpha) \\ Z_j(\mathbf{s}_\beta, t_\beta) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_i^2 & Cij(\mathbf{h}, u) \\ Cij(\mathbf{h}, u) & \sigma_j^2 \end{pmatrix} \right)$$

Sub-likelihood

$$l(i, j, \mathbf{s}_\alpha, \mathbf{s}_\beta, t_\alpha, t_\beta; \theta) = \frac{1}{2} \left\{ \log \Delta_{ij,\alpha\beta} + \frac{A_{ij,\alpha\beta}}{\Delta_{ij,\alpha\beta}} \right\}$$

where $\mathbf{h} = \|\mathbf{s}_\alpha - \mathbf{s}_\beta\|, u = |t_\alpha - t_\beta|, \Delta_{ij,\alpha\beta} = \sigma_i^2 \sigma_j^2 - C_{ij}(\mathbf{h}, u)^2$ and $A_{ij,\alpha\beta} = \sigma_j^2 Z_i(\mathbf{s}_\alpha, t_\alpha)^2 - 2 C_{ij}(\mathbf{h}, u) Z_i(\mathbf{s}_\alpha, t_\alpha) Z_j(\mathbf{s}_\beta, t_\beta) + \sigma_i^2 Z_j(\mathbf{s}_\beta, t_\beta)^2$.

Weighted pairwise likelihood

Weighted pairwise likelihood

$$\mathrm{wpl}(\theta) = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{\alpha=1}^{n} \sum_{\beta>\alpha}^{n} l(i,j,\mathbf{s}_\alpha,\mathbf{s}_\beta,t_\alpha,t_\beta;\theta) w_{\alpha\beta}$$

with $w_{\alpha\beta} = \begin{cases} 1 & \text{if } \mathbf{h} \leq \mathbf{d}_S \text{ and } u \leq d_T \\ 0 & \text{otherwise} \end{cases}$

Bevilacqua et al. (2012)

$$\mathbf{d}^* = \arg \min_{\mathbf{d} \in \mathbb{R}^{d+1}} \mathrm{tr}\left(\mathbf{G}_{np}^{-1}(\mathbf{d};\theta)\right)$$

$\mathbf{d} = (\mathbf{d}_S, d_T)$ and $\mathbf{G}(\theta)$ is the Godambe information matrix (Godambe, 1960)

Problem : requires the calculation of $\nabla wpl(\theta)$

Weighted pairwise likelihood

Weighted pairwise likelihood

$$\mathrm{wpl}(\theta) = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{\alpha=1}^{n} \sum_{\beta>\alpha}^{n} l(i,j,\mathbf{s}_\alpha,\mathbf{s}_\beta,t_\alpha,t_\beta;\theta) w_{\alpha\beta}$$

with $w_{\alpha\beta} = \begin{cases} 1 & \text{if } \mathbf{h} \leq \mathbf{d}_S \text{ and } u \leq d_T \\ 0 & \text{otherwise} \end{cases}$

Bevilacqua et al. (2012)

$$\mathbf{d}^* = \arg \min_{\mathbf{d} \in \mathbb{R}^{d+1}} \mathrm{tr}\left(\mathbf{G}_{np}^{-1}(\mathbf{d};\theta)\right)$$

$\mathbf{d} = (\mathbf{d}_S, d_T)$ and $\mathbf{G}(\theta)$ is the Godambe information matrix (Godambe, 1960)

Problem : requires the calculation of $\nabla wpl(\theta)$

Weighted pairwise likelihood

Weighted pairwise likelihood

$$\mathrm{wpl}(\theta) = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{\alpha=1}^{n} \sum_{\beta>\alpha}^{n} l(i,j,\mathbf{s}_\alpha,\mathbf{s}_\beta,t_\alpha,t_\beta;\theta) w_{\alpha\beta}$$

with $w_{\alpha\beta} = \begin{cases} 1 & \text{if } \mathbf{h} \leq \mathbf{d}_S \text{ and } u \leq d_T \\ 0 & \text{otherwise} \end{cases}$

Bevilacqua et al. (2012)

$$\mathbf{d}^* = \arg \min_{\mathbf{d} \in \mathbb{R}^{d+1}} \mathrm{tr}\left(\mathbf{G}_{np}^{-1}(\mathbf{d};\theta)\right)$$

$\mathbf{d} = (\mathbf{d}_S, d_T)$ and $\mathbf{G}(\theta)$ is the Godambe information matrix (Godambe, 1960)

Problem : requires the calculation of $\nabla wpl(\theta)$

Computation aspects

- $dim(\theta) = \frac{1}{2}(p+2)(p+3)$ where p is the number of variables ;
- To search relevant initial values $\longrightarrow$ $C_{ii}(\mathbf{h},0)$ & $C_{ii}(\mathbf{0},u)$
- Computation on a grid
- Use of profiled likelihood $\longrightarrow$ block of maximum 6 parameters
- Several "for" loops $\longrightarrow$ Use of language C instead of R

Computation aspects

- $dim(\theta) = \frac{1}{2}(p+2)(p+3)$ where p is the number of variables ;
- To search relevant initial values $\longrightarrow$ $C_{ii}(\mathbf{h},0)$ & $C_{ii}(\mathbf{0},u)$
- Computation on a grid
- Use of profiled likelihood $\longrightarrow$ block of maximum 6 parameters
- Several "for" loops $\longrightarrow$ Use of language C instead of R

Computation aspects

- $dim(\theta) = \frac{1}{2}(p+2)(p+3)$ where p is the number of variables ;
- To search relevant initial values $\longrightarrow$ $C_{ii}(\mathbf{h},0)$ & $C_{ii}(\mathbf{0},u)$
- Computation on a grid
- Use of profiled likelihood $\longrightarrow$ block of maximum 6 parameters
- Several "for" loops $\longrightarrow$ Use of language C instead of R

Computation aspects

- $dim(\theta) = \frac{1}{2}(p+2)(p+3)$ where p is the number of variables ;
- To search relevant initial values $\longrightarrow$ $C_{ii}(\mathbf{h},0)$ & $C_{ii}(\mathbf{0},u)$
- Computation on a grid
- Use of profiled likelihood $\longrightarrow$ block of maximum 6 parameters
- Several "for" loops $\longrightarrow$ Use of language C instead of R

Computation aspects

- $dim(\theta) = \frac{1}{2}(p+2)(p+3)$ where p is the number of variables ;
- To search relevant initial values $\longrightarrow$ $C_{ii}(\mathbf{h}, 0)$ & $C_{ii}(\mathbf{0}, u)$
- Computation on a grid
- Use of profiled likelihood $\longrightarrow$ block of maximum 6 parameters
- Several "for" loops $\longrightarrow$ Use of language C instead of R

Outline

Weighting

The inverse of the Godambe matrix is an approximation of the asymptotic variance of the WPL estimator.

$\mathbf{d}$	$d_T = 2$	$d_T = 5$	$d_T = 10$
$d_S = 250$	0.203	0.282	0.287
$d_S = 500$	0.176	0.217	0.260
$d_S = 750$	0.186	0.234	0.297

Table : *Sum of the estimated variance for the 15 parameters over the 100 simulations for each case of* $\mathbf{d}$.

Moreover, the estimated variance of the separability parameter is minimum for $\mathbf{d} = (500, 2)$.

Weighting

The inverse of the Godambe matrix is an approximation of the asymptotic variance of the WPL estimator.

$\mathbf{d}$	$d_T = 2$	$d_T = 5$	$d_T = 10$
$d_S = 250$	0.203	0.282	0.287
$d_S = 500$	0.176	0.217	0.260
$d_S = 750$	0.186	0.234	0.297

Table : *Sum of the estimated variance for the 15 parameters over the 100 simulations for each case of* $\mathbf{d}$.

Moreover, the estimated variance of the separability parameter is minimum for $\mathbf{d} = (500, 2)$.

Weighting

The inverse of the Godambe matrix is an approximation of the asymptotic variance of the WPL estimator.

d	$d_T = 2$	$d_T = 5$	$d_T = 10$
$d_S = 250$	0.203	0.282	0.287
$d_S = 500$	0.176	0.217	0.260
$d_S = 750$	0.186	0.234	0.297

Table : *Sum of the estimated variance for the 15 parameters over the 100 simulations for each case of* $\mathbf{d}$.

Moreover, the estimated variance of the separability parameter is minimum for $\mathbf{d} = (500, 2)$.

Estimation

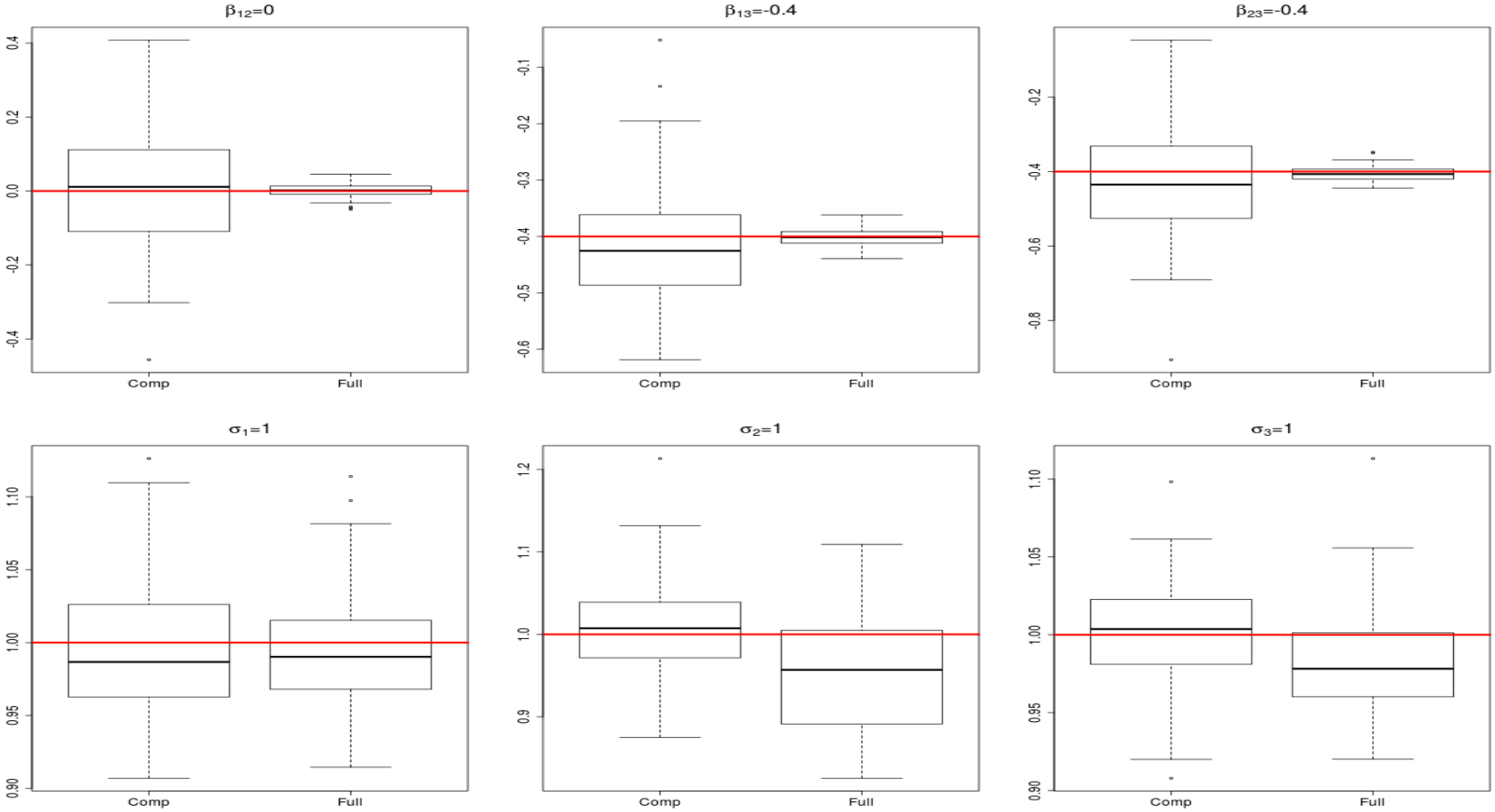

Figure : *Boxplots of the maximum full likelihood estimates and the maximum pairwise likelihood estimates for the 100 simulations.*

Estimation

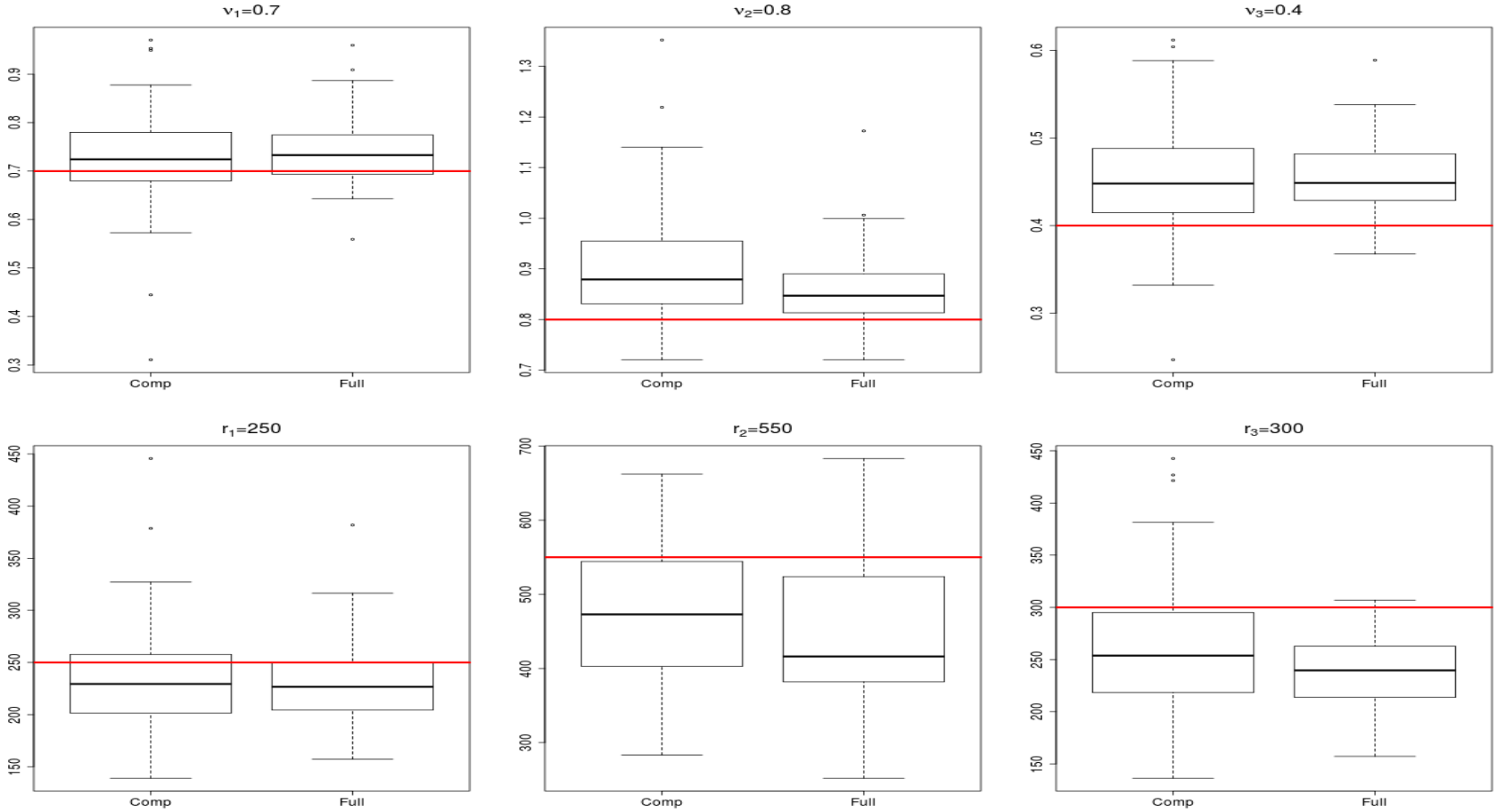

Figure : *Boxplots of the maximum full likelihood estimates and the maximum pairwise likelihood estimates for the 100 simulations.*

Estimation

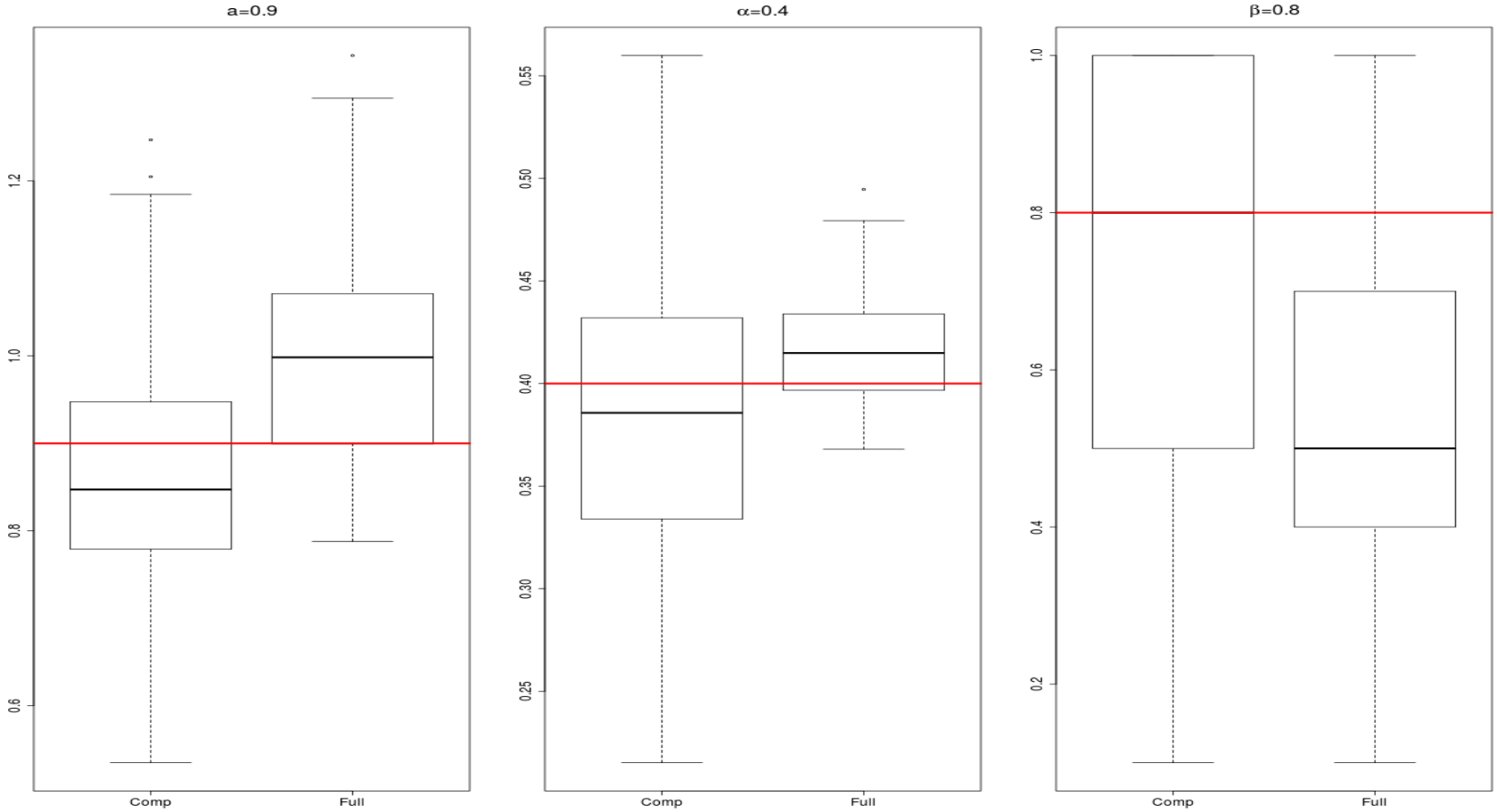

Figure : *Boxplots of the maximum full likelihood estimates and the maximum pairwise likelihood estimates for the 100 simulations.*

Estimation

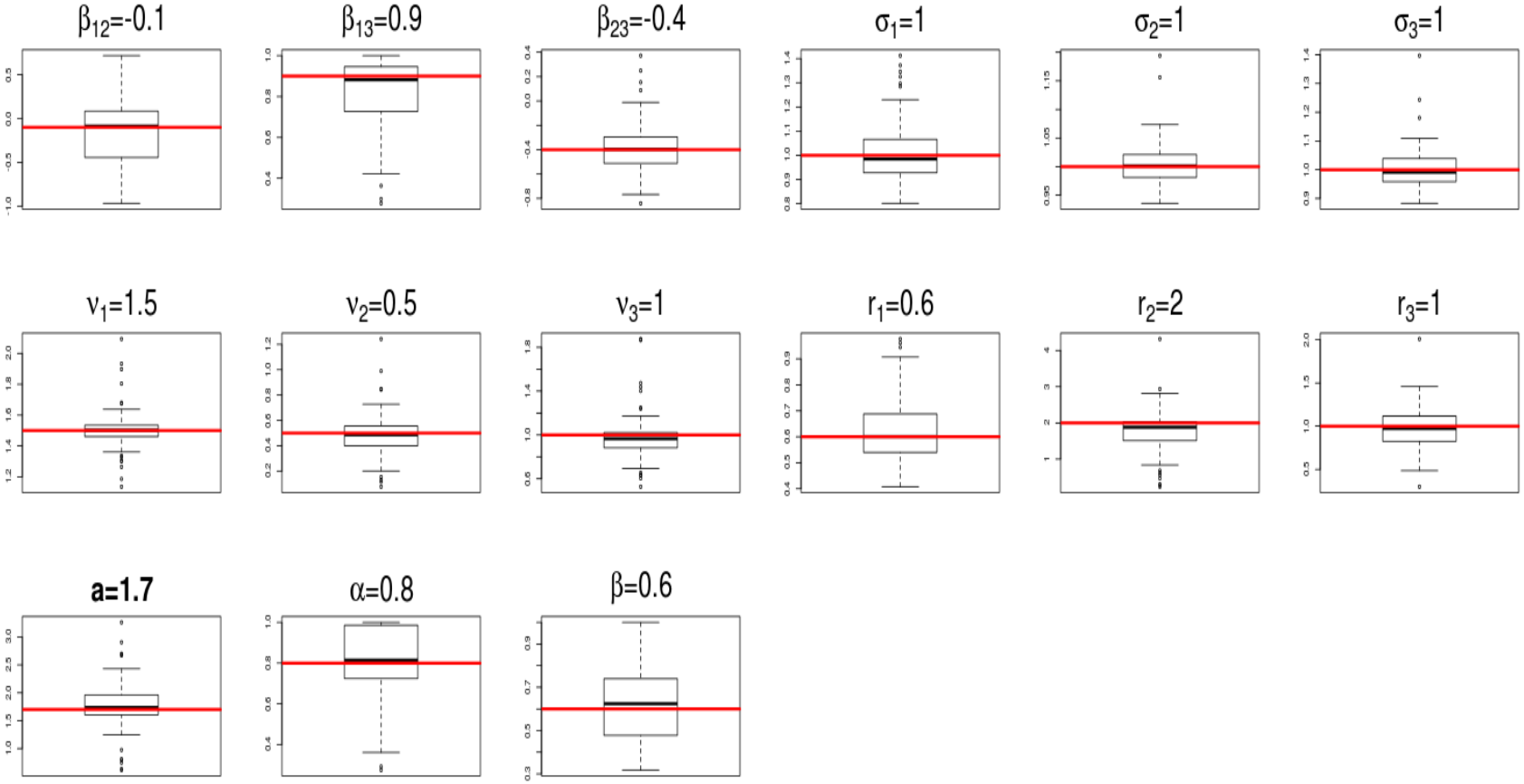

Figure : *Boxplots of the maximum pairwise likelihood estimates for the 200 simulations with 50 locations.*

Validation

Conditional simulation :

$$t-2 : \{Z_1(\mathbf{s}_1, t-2), ..., Z_3(\mathbf{s}_{11}, t-2), Z_1(\mathbf{s}_{12}, t-2), ..., Z_3(\mathbf{s}_{13}, t-2)\}^\mathsf{T}$$
$$t-1 : \{Z_1(\mathbf{s}_1, t-1), ..., Z_3(\mathbf{s}_{11}, t-1), Z_1(\mathbf{s}_{12}, t-1), ..., Z_3(\mathbf{s}_{13}, t-1)\}^\mathsf{T}$$
$$t : \{Z_1(\mathbf{s}_1, t), ..., Z_3(\mathbf{s}_{11}, t), \mathbf{Z}_1(\mathbf{s}_{12}, \mathbf{t}), ..., \mathbf{Z}_3(\mathbf{s}_{13}, \mathbf{t})\}^\mathsf{T}$$

Scores :

- Mean Squared Error (MSE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} \left(Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)\right)^2$
- Mean Absolute Error (MAE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} |Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)|$
- Conitnuous Ranked Probability Score (CRPS) :
  $CRPS(F, x) = \int_{-\infty}^{\infty} (F(y) - \mathbf{1}(y \geq x))^2 \, \mathrm{d}y$

Validation

Conditional simulation :

$$t-2 : \{Z_1(\mathbf{s}_1, t-2), ..., Z_3(\mathbf{s}_{11}, t-2), Z_1(\mathbf{s}_{12}, t-2), ..., Z_3(\mathbf{s}_{13}, t-2)\}^\mathsf{T}$$
$$t-1 : \{Z_1(\mathbf{s}_1, t-1), ..., Z_3(\mathbf{s}_{11}, t-1), Z_1(\mathbf{s}_{12}, t-1), ..., Z_3(\mathbf{s}_{13}, t-1)\}^\mathsf{T}$$
$$t : \{Z_1(\mathbf{s}_1, t), ..., Z_3(\mathbf{s}_{11}, t), \mathbf{Z}_1(\mathbf{s}_{12}, \mathbf{t}), ..., \mathbf{Z}_3(\mathbf{s}_{13}, \mathbf{t})\}^\mathsf{T}$$

Scores :

- Mean Squared Error (MSE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} \left(Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)\right)^2$
- Mean Absolute Error (MAE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} |Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)|$
- Conitnuous Ranked Probability Score (CRPS) :
  $CRPS(F, x) = \int_{-\infty}^{\infty} (F(y) - \mathbf{1}(y \geq x))^2 \, \mathrm{d}y$

Validation

Conditional simulation :

$$t-2 : \{Z_1(\mathbf{s}_1, t-2), ..., Z_3(\mathbf{s}_{11}, t-2), Z_1(\mathbf{s}_{12}, t-2), ..., Z_3(\mathbf{s}_{13}, t-2)\}^\mathsf{T}$$
$$t-1 : \{Z_1(\mathbf{s}_1, t-1), ..., Z_3(\mathbf{s}_{11}, t-1), Z_1(\mathbf{s}_{12}, t-1), ..., Z_3(\mathbf{s}_{13}, t-1)\}^\mathsf{T}$$
$$t : \{Z_1(\mathbf{s}_1, t), ..., Z_3(\mathbf{s}_{11}, t), \mathbf{Z}_1(\mathbf{s}_{12}, \mathbf{t}), ..., \mathbf{Z}_3(\mathbf{s}_{13}, \mathbf{t})\}^\mathsf{T}$$

Scores :

- Mean Squared Error (MSE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} \left(Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)\right)^2$
- Mean Absolute Error (MAE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} |Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)|$
- Conitnuous Ranked Probability Score (CRPS) :
  $CRPS(F, x) = \int_{-\infty}^{\infty} (F(y) - \mathbf{1}(y \geq x))^2 \, \mathrm{d}y$

Validation

Conditional simulation :

$$t-2 : \{Z_1(\mathbf{s}_1, t-2), ..., Z_3(\mathbf{s}_{11}, t-2), Z_1(\mathbf{s}_{12}, t-2), ..., Z_3(\mathbf{s}_{13}, t-2)\}^\mathsf{T}$$
$$t-1 : \{Z_1(\mathbf{s}_1, t-1), ..., Z_3(\mathbf{s}_{11}, t-1), Z_1(\mathbf{s}_{12}, t-1), ..., Z_3(\mathbf{s}_{13}, t-1)\}^\mathsf{T}$$
$$t : \{Z_1(\mathbf{s}_1, t), ..., Z_3(\mathbf{s}_{11}, t), \mathbf{Z}_1(\mathbf{s}_{12}, \mathbf{t}), ..., \mathbf{Z}_3(\mathbf{s}_{13}, \mathbf{t})\}^\mathsf{T}$$

Scores :

- Mean Squared Error (MSE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} \left(Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)\right)^2$
- Mean Absolute Error (MAE) :
  $\sum_{i=6}^{30} \sum_{j=12}^{13} \sum_{k=1}^{3} |Z_k(\mathbf{s}_j, t_i) - \hat{Z}_k(\mathbf{s}_j, t_i)|$
- Conitnuous Ranked Probability Score (CRPS) :
  $CRPS(F, x) = \int_{-\infty}^{\infty} (F(y) - \mathbf{1}(y \geq x))^2 \, \mathrm{d}y$

Validation

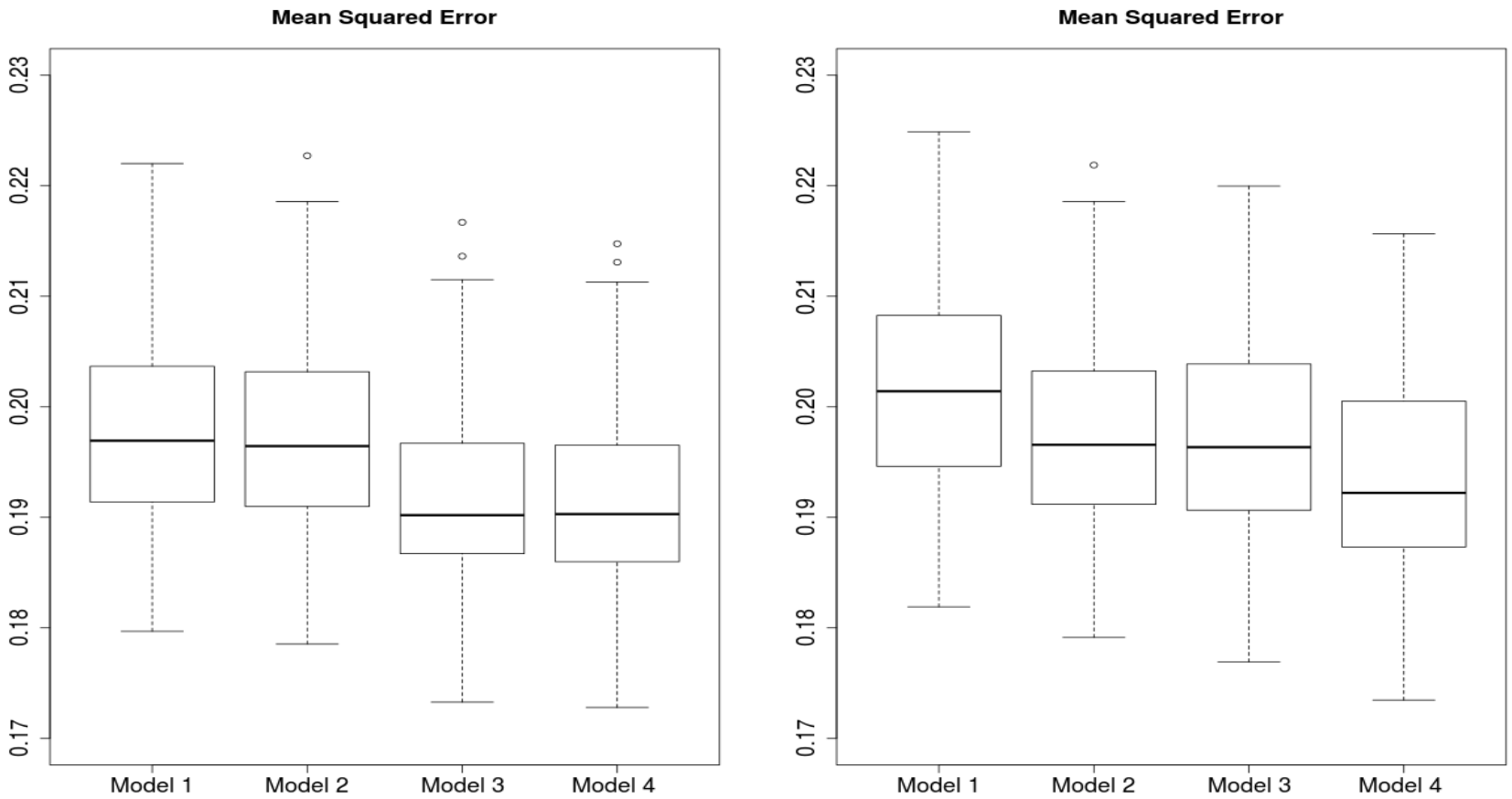

Figure : *Boxplots of the MSE scores over the 100 simulations for the 4 models with the maximum full likelihood estimates (left) and the maximum pairwise likelihood estimates (right).*

Validation

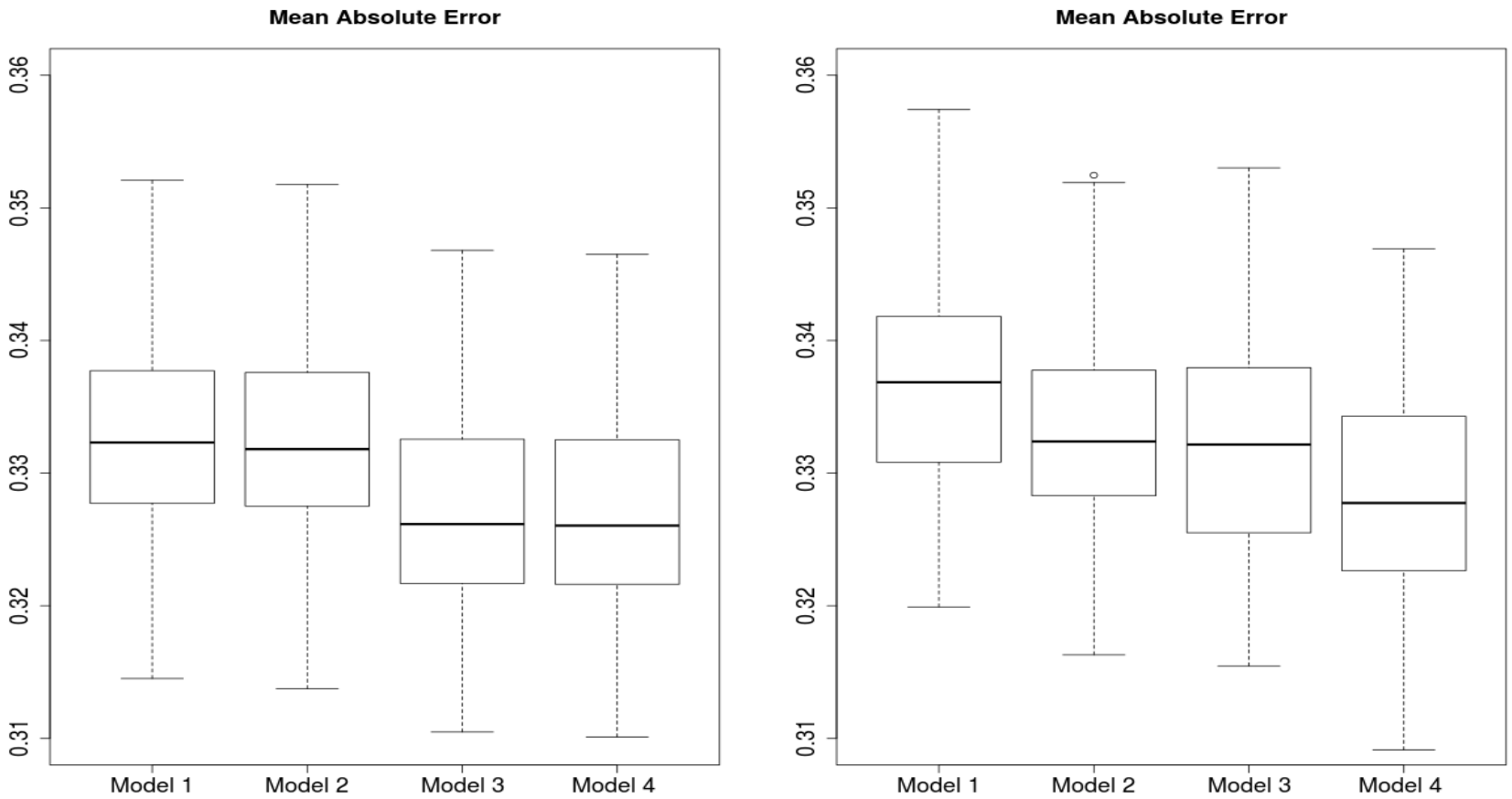

Figure : *Boxplots of the MAE scores over the 100 simulations for the 4 models with the full likelihood estimates (left) and the pairwise likelihood estimates (right).*

Validation

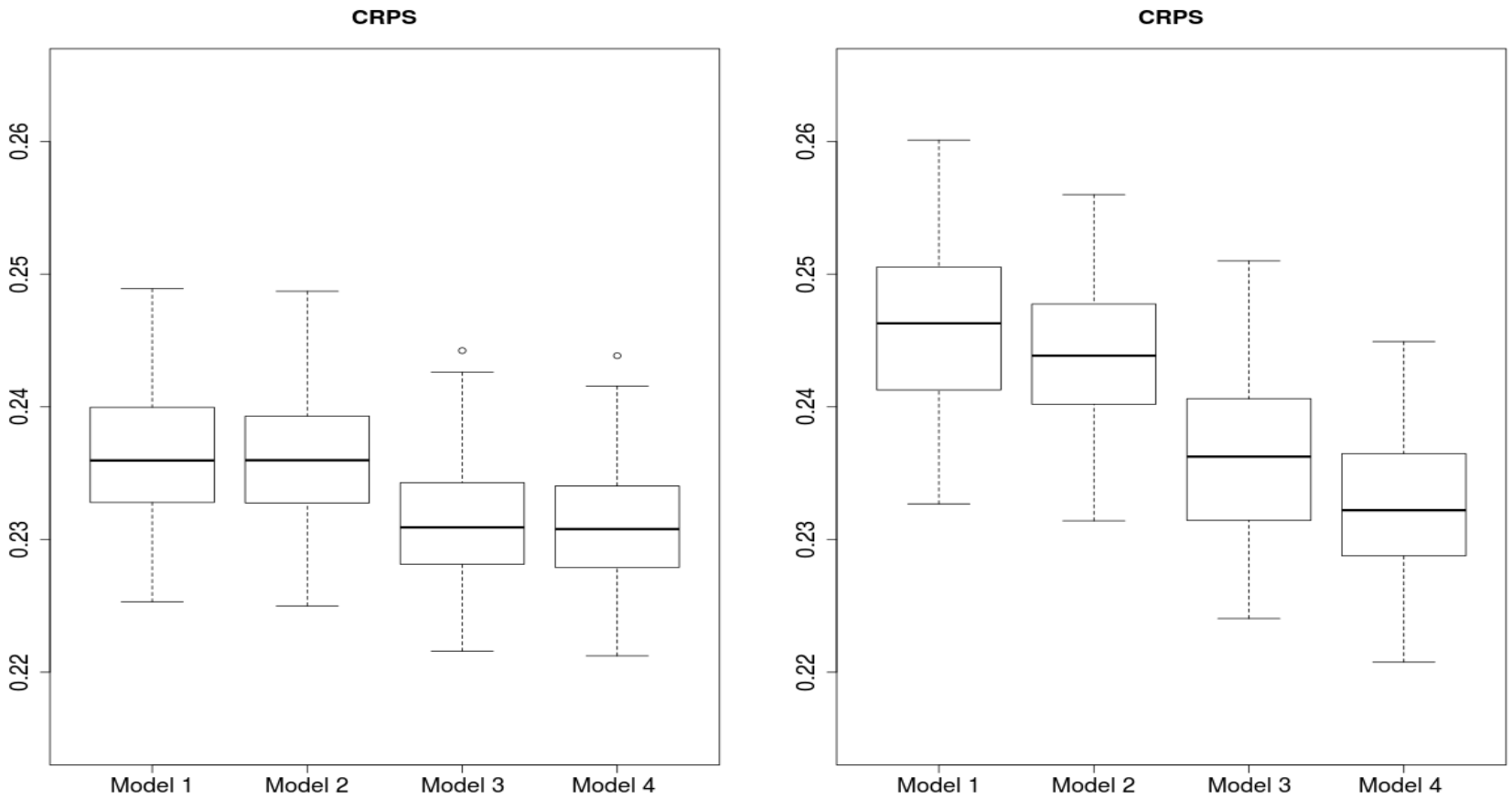

Figure : *Boxplots of the CRPS scores over the 100 simulations for the 4 models with the full likelihood estimates (left) and the pairwise likelihood estimates (right).*

Outline

Conclusion

Méthode d'estimation adaptée

- aux données spatio-temporelles multivariées
- aux modèles de covariance avec beaucoup de paramètres

$\longrightarrow$ Efficace
$\longrightarrow$ Rapide

Nécessite le calibrage de la pondération pour une estimation optimale $\longrightarrow$ calcul de la matrice de Godambe