Supplementary Material: transmission and infected population indicators

Standard mechanistic epidemiological models rely on SIR (Susceptible-Infected-Removed) systems of ordinary differential equations and their extensions (for examples of application to the COVID-19 epidemic, see [5, 6]). Assume here that the dynamics of the epidemic are described by the following SIRD compartmental model:

$$\left\{\begin{array}{l} S'(t) = -\dfrac{\alpha(t)}{N}\, S(t)\, I(t), \\ I'(t) = \dfrac{\alpha(t)}{N}\, S(t)\, I(t) - (\beta+\gamma)\, I(t), \\ R'(t) = \beta\, I(t), \\ D'(t) = \gamma\, I(t), \end{array}\right. \tag{1}$$

with S the susceptible population, I the infectious population, R the recovered population, D the number of deaths due to the epidemic and N the total population (which is assumed to be constant, thereby neglecting the effect of death on the total population size). The time-dependent coefficient $\alpha(t)$ is the contact rate and $1/\beta$ is the mean time until an infectious becomes recovered. The results in [1] indicate that infectiousness starts 2 to 3 days before symptom onset and declines significantly 8 days after symptom onset. Based on these observations we assume here that the mean duration of the infectiousness period is $1/\beta = 10$ days. In [2], the duration of the incubation period was estimated to have a mean of 5.2 days. Thus, the mean duration of the non-infectious exposed period is relatively short (about 2 to 3 days), and can be neglected without much differences on the results, as shown in [3]. The parameter γ corresponds to the death rate of the infectious, which has been estimated in [5, 6], based on French data: $\gamma = 8 \cdot 10^{-4}$ (corresponding to an infection fatality rate of $\gamma/(\gamma+\beta) \approx 0.8\%$).

Computation of R_0 based on the number of observed deaths. To compute the basic reproduction number, we assume that the contact rate α is constant at the beginning of the epidemic, during a period (t_0, t_1). The date t_0 is defined as the first day such that the observed number of deaths $\hat{D}_{t_0}$ exceeds 10, and t_1 is the first date where the number of deaths $\hat{D}_{t_1}$ crosses the value 100. The basic reproduction number can be computed as [4]:

$$R_0 = \frac{\alpha}{\beta+\gamma}.$$

As $S \approx N$ at the early stage of the epidemics and since $\beta + \gamma \approx \beta$, we have:

$$I'(t) = I\,\left(\alpha\,\frac{S}{N} - (\beta+\gamma)\right) \approx I\,\beta\,(R_0 - 1),$$

which can be solved explicitly, leading to

$$\left\{\begin{array}{l} I(t) = I_0\, e^{\beta\,(R_0-1)\,(t-t_0)}, \\ R(t) = \dfrac{I_0}{R_0-1}\left(e^{\beta\,(R_0-1)\,(t-t_0)} - 1\right), \\ D(t) = \hat{D}(t_0) + \dfrac{\gamma}{\beta}\dfrac{I_0}{R_0-1}\left(e^{\beta\,(R_0-1)\,(t-t_0)} - 1\right), \end{array}\right.$$

with $R(t_0) = 0$, $I(t_0) = I_0$ and $D(t_0) = \hat{D}_{t_0}$.

Fixing the value $\hat{D}_{t_1}$ (cumulated number of recorded deaths on day t_1), we get:

$$I_0 = \frac{\beta}{\gamma}\frac{(R_0-1)(\hat{D}_{t_1} - \hat{D}_{t_0})}{e^{\beta\,(R_0-1)\,(t_1-t_0)} - 1}.$$

Finally,

$$D_t = \hat{D}_{t_0} + (\hat{D}_{t_1} - \hat{D}_{t_0})\frac{e^{\beta\,(R_0-1)\,(t-t_0)} - 1}{e^{\beta\,(R_0-1)\,(t_1-t_0)} - 1}, \tag{2}$$

for all $t \in (t_0, t_1)$. R_0 is then computed by fitting the above formula to the data $\hat{D}_t$ with a standard nonlinear curve fitting procedure (using Matlab® Curve Fitting Toolbox®), over the period (t_0, t_1).

Computation of R_t based on the number of observed deaths. The effective reproduction number is defined as:

$$R_t = \frac{\alpha}{\beta+\gamma}\frac{S(t)}{N}.$$

The value of R_t is computed with the same procedure as R_0 (we use the same approximations as above: $S \approx N$ and $\beta+\gamma \approx \beta$), but instead of fitting (2) over a fixed period (t_0, t_1), we apply the fitting procedure over a dynamical time period $(t-\tau, t)$. Thus, R_t is computed by fitting to the data $\hat{D}_s$ for $s \in (t-\tau, t)$ (same fitting procedure as above) the formula:

$$D_s = \hat{D}_{t-\tau} + (\hat{D}_t - \hat{D}_{t-\tau})\frac{e^{\beta\,(R_t-1)\,(s-(t-\tau))}-1}{e^{\beta\,\tau\,(R_t-1)}-1}. \tag{3}$$

In the illustrations presented in the main text, the size of the window was $\tau = 15$ days.

Computation of the actual number of cases. We use the above procedure to compute R_t, and to deduce $\alpha(t) \approx \beta\, R_t$. The computation of the number of cases begins at some time t_0 defined again as the first day such that the observed number of deaths $\hat{D}_{t_0}$ exceeds 10. To initialise the number of infectious $I_0 = I(t_0)$, we use the equation $D'(t) = \gamma\, I(t)$, and we define I_0 as $1/\gamma \times$ (mean number of deaths over the period ranging from 5 days before t_0 to 5 days after t_0). $S(t_0) = N$ corresponds to the population size in the considered country.

Then, the solution of the system (1) can be computed with a standard numerical algorithm, using e.g. Matlab® *ode45* solver. The daily number of cases is computed as $S(t-1) - S(t)$, the variation in the number of susceptibles between two consecutive days.

References

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